CHAPTER 14

Dc Drives

After completing this chapter, students should be able to do the following:

- Describe the basic characteristics of dc motors and their control characteristics.
- List the types of dc drives and their operating modes.
- List the control requirements of four-quadrant drives.
- Describe the parameters of the transfer function of converter-fed dc motors.
- Determine the performance parameters of single-phase and three-phase converter drives.
- Determine the performance parameters of dc–dc converter drives.
- Determine the closed-loop and open-loop transfer functions of dc motors.
- Determine the speed and torque characteristics of converter-fed drives.
- Design and analyze a feedback control of a motor drive.
- Determine the optimized parameters of the current- and speed feedback controller.

Symbols and Their Meanings

Symbols	Meaning
$\alpha_a ; \alpha_f$	Delay angle of the armature and field circuit converter, respectively
$\tau_a; \tau_f; \tau_m;$	Armature, field, and mechanical time constants, respectively
$\omega; \omega_o$	Normal and no-load motor speeds, respectively
$B; J$	Viscous friction and inertia of a motor, respectively
$e_g; E_g$	Instantaneous and the average back emf of a dc motor, respectively
$f; f_s$	Switching frequency of a dc–dc converter and supply frequency, respectively
$i_a; I_a$	Instantaneous and the average armature motor current, respectively
$i_f; I_f$	Instantaneous and the average field motor current, respectively
I_s	Average supply current
$K_t; K_v; K_b$	Torque constant, generator constant, and back emf constant, respectively
$K_r; \tau_r$	Gain and the time constant of the converter, respectively
$K_c; \tau_c$	Gain and the time constant of the current controller, respectively
$K_s; \tau_s$	Gain and the time constant of the speed controller, respectively
$K_\omega ; \tau_\omega$	Gain and the time constant of the speed feedback filter, respectively
$L_a; L_f$	Armature and field circuit inductance of a dc motor, respectively
$L_m; R_m$	Motor inductance and resistance, respectively

(*continued*)

Symbols	Meaning
PF	Input power factor of a converter
P_i; P_o	Input and output powers of a converter, respectively
P_d;P_g	Average developed power and regenerated power of a motor, respectively
P_b; V_b	Power and voltage of a braking resistance, respectively
R_a; R_f	Armature and field circuit resistances of a dc motor, respectively
R_{eq}	Equivalent resistance offered by a converter
T_d; T_L	Developed torque and load torque, respectively
V_a; V_f	Average armature and field voltages of a motor, respectively

14.1 INTRODUCTION

Direct current (dc) motors have variable characteristics and are used extensively in variable-speed drives. Dc motors can provide a high starting torque and it is also possible to obtain speed control over a wide range. The methods of speed control are normally simpler and less expensive than those of ac drives. Dc motors play a significant role in modern industrial drives. Both series and separately excited dc motors are normally used in variable-speed drives, but series motors are traditionally employed for traction applications. Due to commutators, dc motors are not suitable for very high-speed applications and require more maintenance than do ac motors. With the recent advancements in power conversions, control techniques, and microcomputers, the ac motor drives are becoming increasingly competitive with dc motor drives. Although the future trend is toward ac drives, dc drives are currently used in many industries. It might be a few decades before the dc drives are completely replaced by ac drives.

There are also disadvantages of variable speed drives (VSDs), such as the cost of space, cooling, and capital cost. The VSDs also generate acoustic noise, cause motor derating, and generate supply harmonics. The PWM voltage-source inverter (VSI) drives manufactured with fast-switching devices add other problems such as (a) premature motor insulation failures, (b) bearing/earth current, and (c) electromagnetic compatibility (EMC) issues.

Controlled rectifiers provide a variable dc output voltage from a fixed ac voltage, whereas a dc–dc converter can provide a variable dc voltage from a fixed dc voltage. Due to their ability to supply a continuously variable dc voltage, controlled rectifiers and dc–dc converters made a revolution in modern industrial control equipment and variable-speed drives, with power levels ranging from fractional horsepower to several megawatts. Controlled rectifiers are generally used for the speed control of dc motors, as shown in Figure 14.1a. The alternative form would be a diode rectifier followed by dc–dc converter, as shown in Figure 14.1b. Dc drives can be classified, in general, into three types:

1. Single-phase drives
2. Three-phase drives
3. Dc–dc converter drives

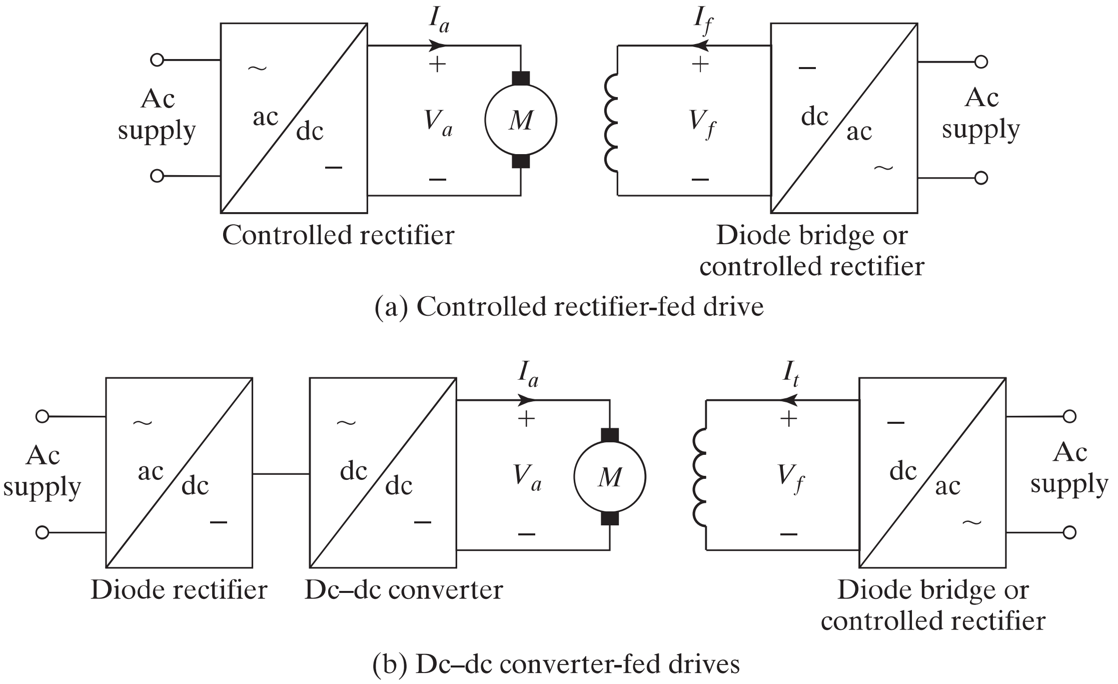

FIGURE 14.1

Controller rectifier- and dc–dc converter-fed drives.

Single-phase drives are used in low-power applications in the range up to 100 kW. Three-phase drives are used for applications in the range 100 kW to 500 kW. The converters are also connected in series and parallel to produce 12-pulses output. The power range can go as high as 1 MW for high-power drives. These drives generally require harmonic filters and their size could be quite bulky [11].

14.2 BASIC CHARACTERISTICS OF DC MOTORS

Dc motors can be classified into two types depending on the type of field winding connections: (i) shunt and (ii) series. In a shunt-field motor, the field excitation is independent of the armature circuit. The field excitation can be controlled independently and this type of motor is often called the *separately excited* motor. That is, the armature and the field currents are different. In a series-type motor, the field excitation circuit is connected in series with the armature circuit. That is, the armature and the field currents are the same.

14.2.1 Separately Excited Dc Motor

The equivalent circuit for a separately excited dc-motor is shown in Figure 14.2 [1]. When a separately excited motor is excited by a field current of i_f and an armature current of i_a flows in the armature circuit, the motor develops a back electromotive force (emf) and a torque to balance the load torque at a particular speed. The field current i_f of a separately excited motor is independent of the armature current i_a and any change in the armature current has no effect on the field current. The field current is normally much less than the armature current.

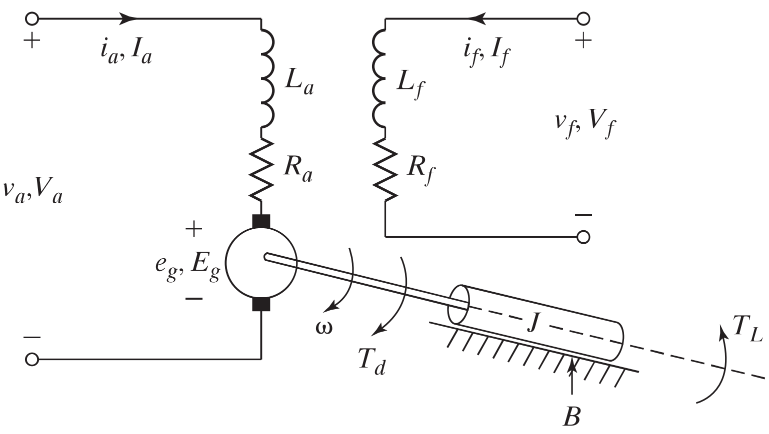

FIGURE 14.2

Equivalent circuit of separately excited dc motors.

The equations describing the characteristics of a separately excited motor can be determined from Figure 14.2. The instantaneous field current i_f is described as

$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

The instantaneous armature current can be found from

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_g$$

The motor back emf, which is also known as *speed voltage*, is expressed as

$$e_g = K_v \omega i_f$$

The torque developed by the motor is

$$T_d = K_t i_f i_a$$

The developed torque must be equal to the load torque:

$$T_d = J\frac{d\omega}{dt} + B\omega + T_L$$

where ω = motor angular speed, or rotor angular frequency, rad/s;
B = viscous friction constant, N · m/rad/s;
K_v = voltage constant, V/A-rad/s;
K_t = torque constant, which equals voltage constant, K_v;
L_a = armature circuit inductance, H;
L_f = field circuit inductance, H;
R_a = armature circuit resistance, Ω;
R_f = field circuit resistance, Ω;
T_L = load torque, N · m.

Under steady-state conditions, the time derivatives in these equations are zero and the steady-state average quantities are

$$V_f = R_f I_f \tag{14.1}$$

$$E_g = K_v \omega I_f \tag{14.2}$$

$$V_a = R_a I_a + E_g$$

$$= R_a I_a + K_v \omega I_f \tag{14.3}$$

$$T_d = K_t I_f I_a \tag{14.4}$$

$$= B\omega + T_L \tag{14.5}$$

The developed power is

$$P_d = T_d \omega \tag{14.6}$$

The relationship between the field current I_f and the back emf E_g is nonlinear due to magnetic saturation. The relationship, which is shown in Figure 14.3, is known as *magnetization characteristic* of the motor. From Eq. (14.3), the speed of a separately excited motor can be found from

$$\omega = \frac{V_a - R_a I_a}{K_v I_f} = \frac{V_a - R_a I_a}{K_v V_f / R_f} \tag{14.7}$$

We can notice from Eq. (14.7) that the motor speed can be varied by (1) controlling the armature voltage V_a, known as *voltage control*; (2) controlling the field current I_f, known as *field control*; or (3) torque demand, which corresponds to an armature current I_a, for a fixed field current I_f. The speed, which corresponds to the rated armature voltage, rated field current, and rated armature current, is known as the *rated* (or *base*) *speed*.

In practice, for a speed less than the base speed, the armature current and field currents are maintained constant to meet the torque demand, and the armature voltage V_a is varied to control the speed. For speed higher than the base speed, the armature voltage is maintained at the rated value and the field current is varied to control the speed. However, the power developed by the motor $(= \text{torque} \times \text{speed})$ remains constant. Figure 14.4 shows the characteristics of torque, power, armature current, and field current against the speed.

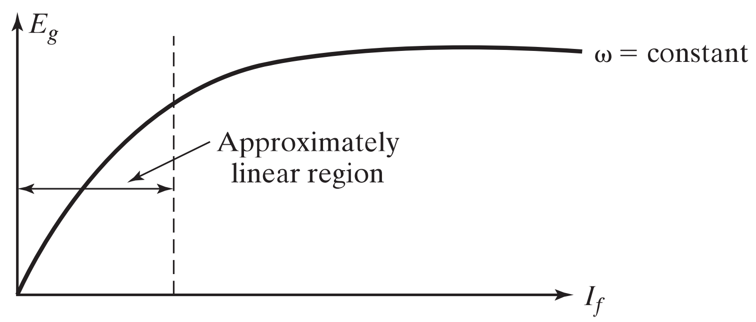

FIGURE 14.3

Magnetization characteristic.

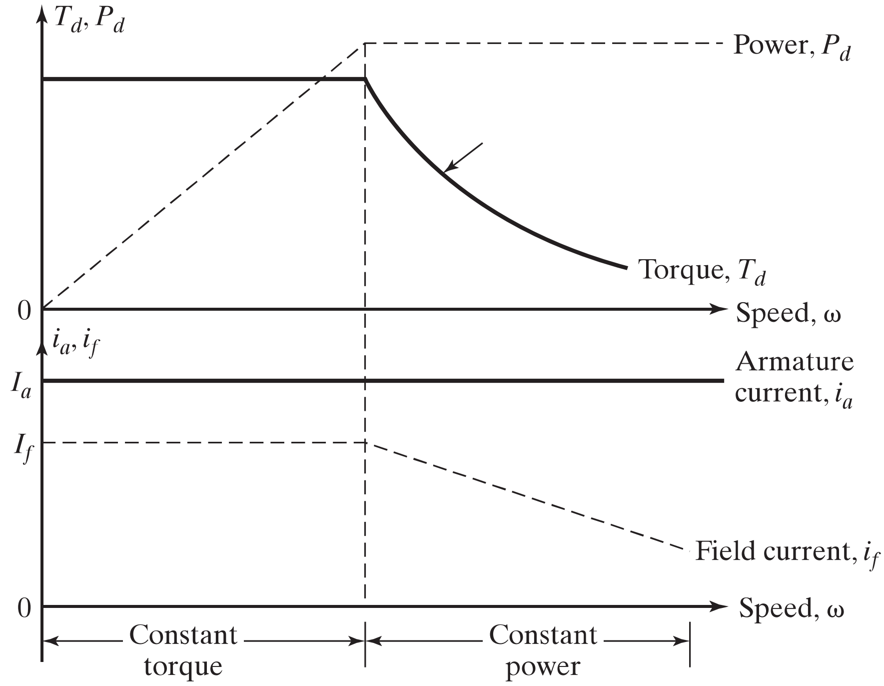

FIGURE 14.4
Characteristics of separately excited motors.

14.2.2 Series-Excited Dc Motor

The field of a dc motor may be connected in series with the armature circuit, as shown in Figure 14.5, and this type of motor is called a *series motor*. The field circuit is designed to carry the armature current. The steady-state average quantities are

$$E_g = K_v \omega I_a \tag{14.8}$$

$$V_a = (R_a + R_f) I_a + E_g \tag{14.9}$$

$$= (R_a + R_f) I_a + K_v \omega I_f \tag{14.10}$$

$$T_d = K_t I_a I_f$$

$$= B\omega + T_L \tag{14.11}$$

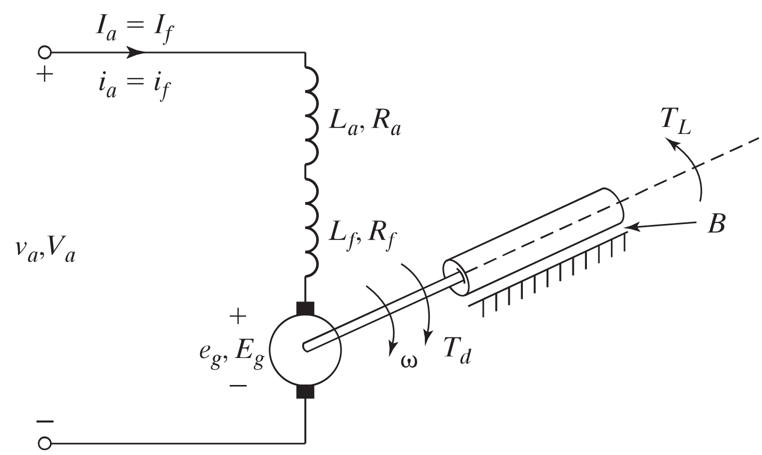

FIGURE 14.5
Equivalent circuit of dc series motors.

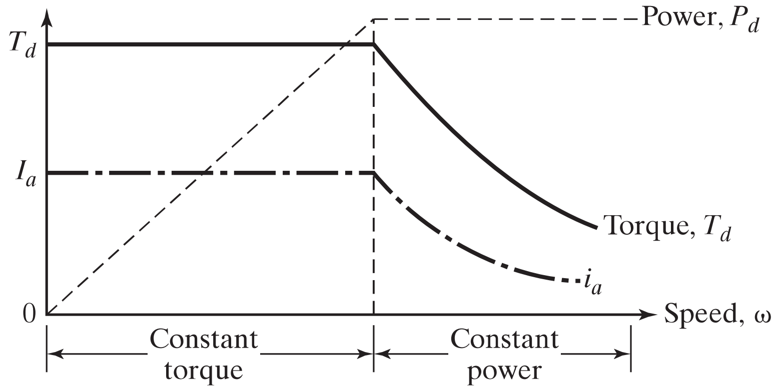

FIGURE 14.6
Characteristics of dc series motors.

The speed of a series motor can be determined from Eq. (14.10):

$$\omega = \frac{V_a - (R_a + R_f)I_a}{K_v I_f} \tag{14.12}$$

The speed can be varied by controlling the (1) armature voltage V_a; or (2) armature current, which is a measure of the torque demand. Equation (14.11) indicates that a series motor can provide a high torque, especially at starting; for this reason, series motors are commonly used in traction applications.

For a speed up to the base speed, the armature voltage is varied and the torque is maintained constant. Once the rated armature voltage is applied, the speed–torque relationship follows the natural characteristic of the motor and the power (= torque × speed) remains constant. As the torque demand is reduced, the speed increases. At a very light load, the speed could be very high and it is not advisable to run a dc series motor without a load. Figure 14.6 shows the characteristics of dc series motors.

Example 14.1 Finding the Voltage and Current of a Separately Excited Motor

A 15-hp, 220-V, 2000-rpm separately excited dc motor controls a load requiring a torque of $T_L = 45$ N·m at a speed of 1200 rpm. The field circuit resistance is $R_f = 147\ \Omega$, the armature circuit resistance is $R_a = 0.25\ \Omega$, and the voltage constant of the motor is $K_v = 0.7032$ V/A rad/s. The field voltage is $V_f = 220$ V. The viscous friction and no-load losses are negligible. The armature current may be assumed continuous and ripple free. Determine (a) the back emf E_g, (b) the required armature voltage V_a, and (c) the rated armature current of the motor.

Solution

$R_f = 147\ \Omega$, $R_a = 0.25\ \Omega$, $K_v = K_t = 0.7032$ V/A rad/s, $V_f = 220$ V, $T_d = T_L = 45$ N·m, $\omega = 1200\ \pi/30 = 125.66$ rad/s, and $I_f = 220/147 = 1.497$ A.

a. From Eq. (14.4), $I_a = 45/(0.7032 \times 1.497) = 42.75$ A. From Eq. (14.2), $E_g = 0.7032 \times 125.66 \times 1.497 = 132.28$ V.

b. From Eq. (14.3), $V_a = 0.25 \times 42.75 + 132.28 = 142.97$ V.

c. Because 1 hp is equal to 746 W, $I_{\text{rated}} = 15 \times 746/220 = 50.87$ A.

14.2.3 Gear Ratio

In general, the load torque is a function of speed. For example, the load torque is proportional to speed in frictional systems such as a feed drive. In fans and pumps, the load torque is proportional to the square of the speed. The motor is often connected to the load through a set of gears. The gears have a teeth ratio and can be treated as torque transformers, as shown in Figure 14.7. The gears are primarily used to amplify the torque on the load side that is at a lower speed compared to the motor speed. The motor is designed to run at high speeds because the higher the speed, the lower is the volume and size of the motor. But most of the applications require low speeds and there is a need for a gearbox in the motor–load connection. Assuming zero losses in the gearbox, the power handled by the gear is the same on both sides. That is,

$$T_1 \omega_1 = T_2 \omega_2 \tag{14.13}$$

The speed on each side is inversely proportional to its tooth number. That is,

$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} \tag{14.14}$$

Substituting Eq. (14.14) into Eq. (14.13) gives

$$T_2 = \left(\frac{N_2}{N_1}\right)^2 T_1 \tag{14.15}$$

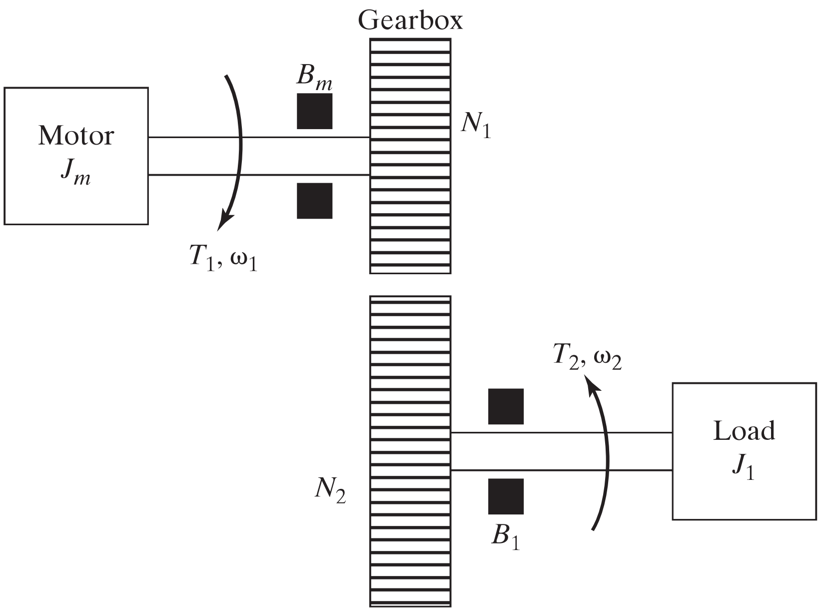

FIGURE 14.7

Schematic for a gearbox between the motor and the load.

Similar to a transformer, the load inertia J_1 and the load bearing constant B_1 can be reflected to the motor side by

$$J = J_m + \left(\frac{N_1}{N_2}\right)^2 J_1 \tag{14.16}$$

$$B = B_m + \left(\frac{N_1}{N_2}\right)^2 B_1 \tag{14.17}$$

where

J_m and J_1 are the motor inertia and the load inertia
B_m and B_1 are the motor side and load side friction coefficients

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Example 14.2 Determining the Effects of Gear Ratio on the Effective Motor Torque and Inertia

The parameters of the gearbox shown in Figure 14.7 are $B_1 = 0.025\,\text{Nm/rad/s}$, $\omega_1 = 210\,\text{rad/s}$, $B_m = 0.045\,\text{kg-m}^2$, $J_m = 0.32\,\text{kg-m}^2$, $T_2 = 20\,\text{Nm}$, and $\omega_2 = 21\,\text{rad/s}$. Determine (a) the gear ratio $GR = N_2/N_1$, (b) the effective motor torque T_1, (c) the effective inertia J, and (d) the effective friction coefficient B.

Solution

$B_1 = 0.025\,\text{Nm/rad/s}$, $\omega_1 = 210\,\text{rad/s}$, $B_m = 0.045\,\text{kg-m}^2$, $J_m = 0.32\,\text{kg-m}^2$, $T_2 = 20\,\text{Nm}$, and $\omega_2 = 21\,\text{rad/s}$.

a. Using Eq. (14.14), $GR = \dfrac{N_2}{N_1} = \dfrac{\omega_1}{\omega_2} = \dfrac{210}{21} = 10$

b. Using Eq. (14.15), $T_1 = \dfrac{T_2}{GR^2} = \dfrac{20}{10^2} = 0.2\,\text{Nm}$

c. Using Eq. (14.16), $J = J_m + \dfrac{J_1}{GR^2} = 0.32 + \dfrac{0.25}{10^2} = 0.323\,\text{kg-m}^2$

d. Using Eq. (14.17), $B = B_m + \dfrac{B_1}{GR^2} = 0.045 + \dfrac{0.025}{10^2} = 0.045\,\text{Nm/rad/s}$

Key Points of Section 14.2

- The speed of a dc motor can be varied by controlling (1) the armature voltage, (2) the field current, or (3) the armature current that is a measure of the torque demand.
- For a speed less than the rated speed (also known as base speed), the armature voltage is varied to control the speed, while the armature and field currents are maintained constant. For a speed higher than the rated speed, the field current is varied to control the speed, while the armature voltage is maintained at the rated value.
- The motor is often connected to the load through a gearbox. The effects of the reflected-load inertia and the load friction coefficient should be included in evaluating the performances of a motor drive.

14.3 OPERATING MODES

In variable-speed applications, a dc motor may be operating in one or more modes: motoring, regenerative braking, dynamic braking, plugging, and four quadrants [2, 3]. The operation of the motor in any one of these modes requires connecting the field and armature circuits in different arrangements, as shown in Figure 14.8. This is done by switching power semiconductor devices and contactors.

Motoring. The arrangements for motoring are shown in Figure 14.8a. Back emf E_g is less than supply voltage V_a. Both armature and field currents are positive. The motor develops torque to meet the load demand.

Regenerative braking. The arrangements for regenerative braking are shown in Figure 14.8b. The motor acts as a generator and develops an induced voltage E_g. E_g must be greater than supply voltage V_a. The armature current is negative, but the field current is positive. The kinetic energy of the motor is returned to the supply. A series motor is usually connected as a self-excited generator. For self-excitation, it is necessary that the field current aids the residual flux. This is normally accomplished by reversing the armature terminals or the field terminals.

Dynamic braking. The arrangements shown in Figure 14.8c are similar to those of regenerative braking, except the supply voltage V_a is replaced by a braking resistance R_b. The kinetic energy of the motor is dissipated in R_b.

Plugging. Plugging is a type of braking. The connections for plugging are shown in Figure 14.8d. The armature terminals are reversed while running. The supply voltage V_a and the induced voltage E_g act in the same direction. The armature current is reversed, thereby producing a braking torque. The field current is positive. For a series motor, either the armature terminals or field terminals should be reversed, but not both.

Four quadrants. Figure 14.9 shows the polarities of the supply voltage V_a, back emf E_g, and armature current I_a for a separately excited motor. In forward motoring (quadrant I), V_a, E_g, and I_a are all positive. The torque and speed are also positive in this quadrant.

During forward braking (quadrant II), the motor runs in the forward direction and the induced emf E_g continues to be positive. For the torque to be negative and the direction of energy flow to reverse, the armature current must be negative. The supply voltage V_a should be kept less than E_g.

In reverse motoring (quadrant III), V_a, E_g, and I_a are all negative. The torque and speed are also negative in this quadrant. To keep the torque negative and the energy flow from the source to the motor, the back emf E_g must satisfy the condition $|V_a| > |E_g|$. The polarity of E_g can be reversed by changing the direction of field current or by reversing the armature terminals.

During reverse braking (quadrant IV), the motor runs in the reverse direction. V_a and E_g continue to be negative. For the torque to be positive and the energy to flow from the motor to the source, the armature current must be positive. The induced emf E_g must satisfy the condition $|V_a| < |E_g|$.

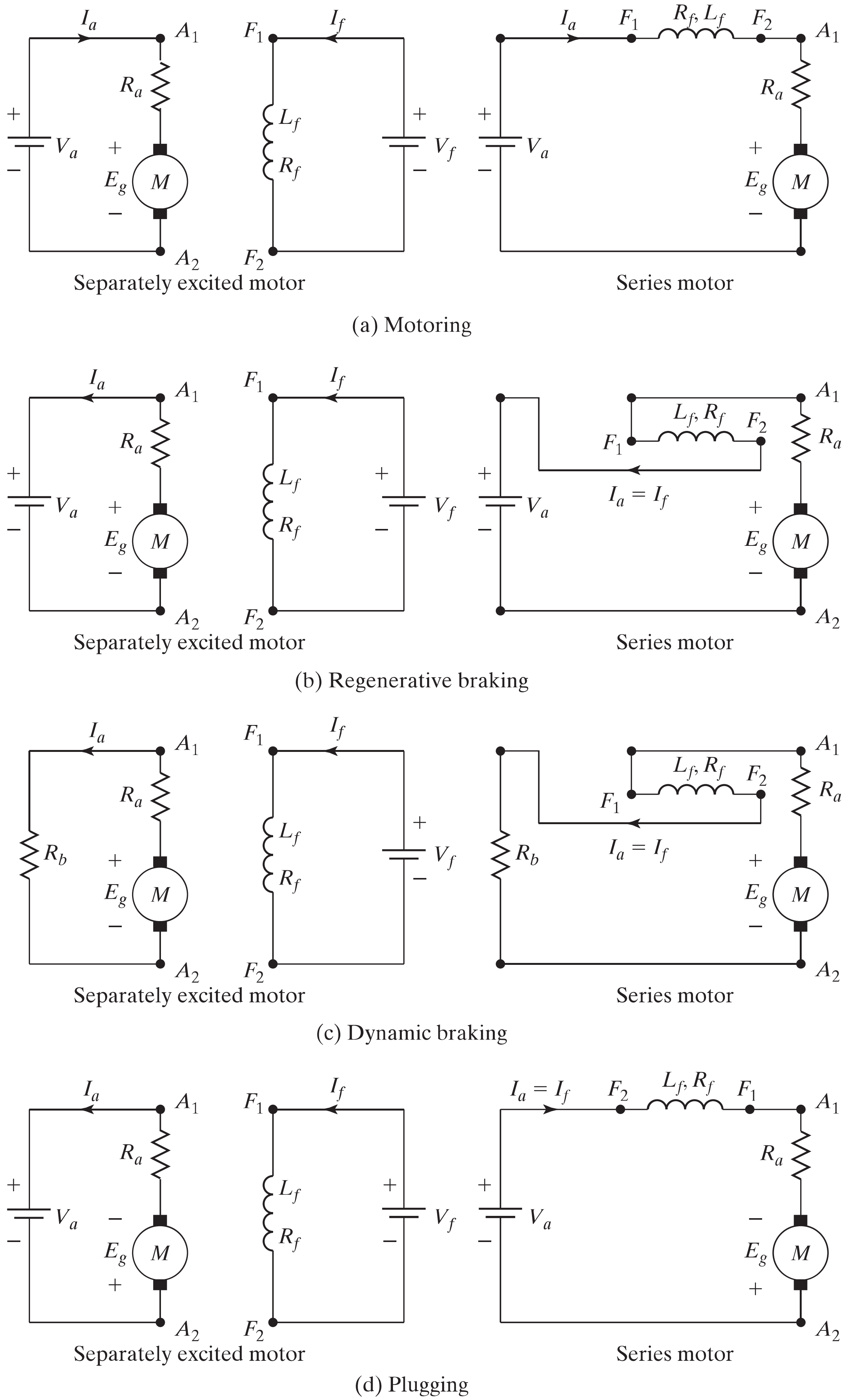


FIGURE 14.8

Operating modes.

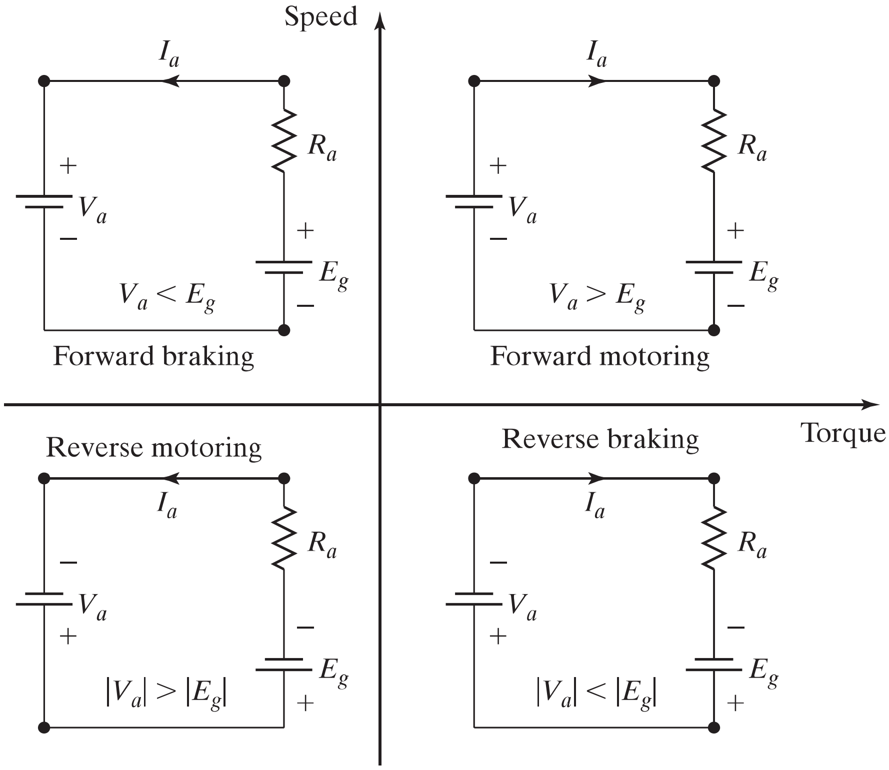


FIGURE 14.9

Conditions for four quadrants.

Key Points of Section 14.3

- A motor drive should be capable of four quadrant operations: forward motoring, forward braking, reverse motoring, or reverse braking.
- For operations in the reverse direction, the field excitation must be reversed to reverse the polarity of the back emf.

14.4 SINGLE-PHASE DRIVES

If the armature circuit of a dc motor is connected to the output of a single-phase controlled rectifier, the armature voltage can be varied by varying the delay angle of the converter α_a. The forced-commutated ac–dc converters can also be used to improve the power factor (PF) and to reduce the harmonics. The basic circuit agreement for a single-phase converter-fed separately excited motor is shown in Figure 14.10. At a low delay angle, the armature current may be discontinuous, and this would increase the losses in the motor. A smoothing inductor, L_m, is normally connected in series with the armature circuit to reduce the ripple current to an acceptable magnitude. A converter is also applied in the field circuit to control the field current by varying the delay angle α_f. For operating the motor in a particular mode, it is often necessary to use contactors for reversing the armature circuit, as shown in Figure 14.11a, or the field circuit, as shown in Figure 14.11b. To avoid inductive voltage surges, the field or the armature reversing is performed at a zero armature current. The delay (or firing) angle is normally adjusted to give a zero current; additionally, a dead time of typically

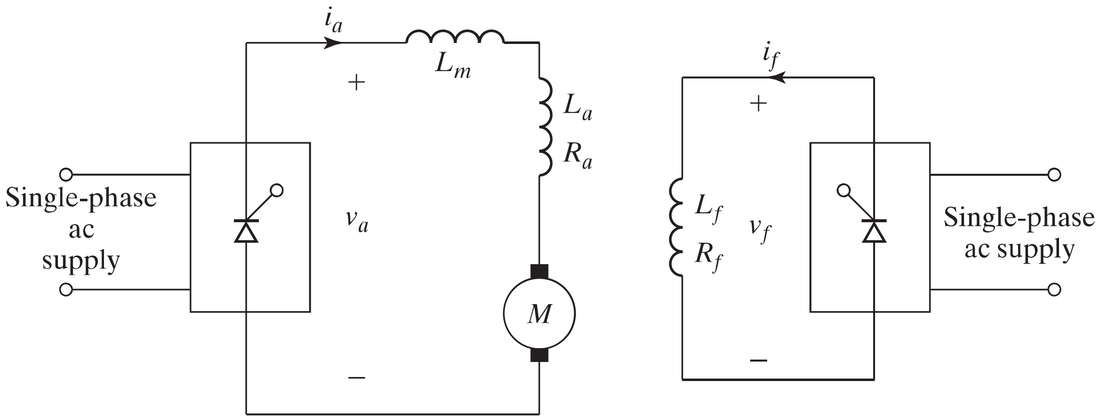


FIGURE 14.10

Basic circuit arrangement of a single-phase dc drive.

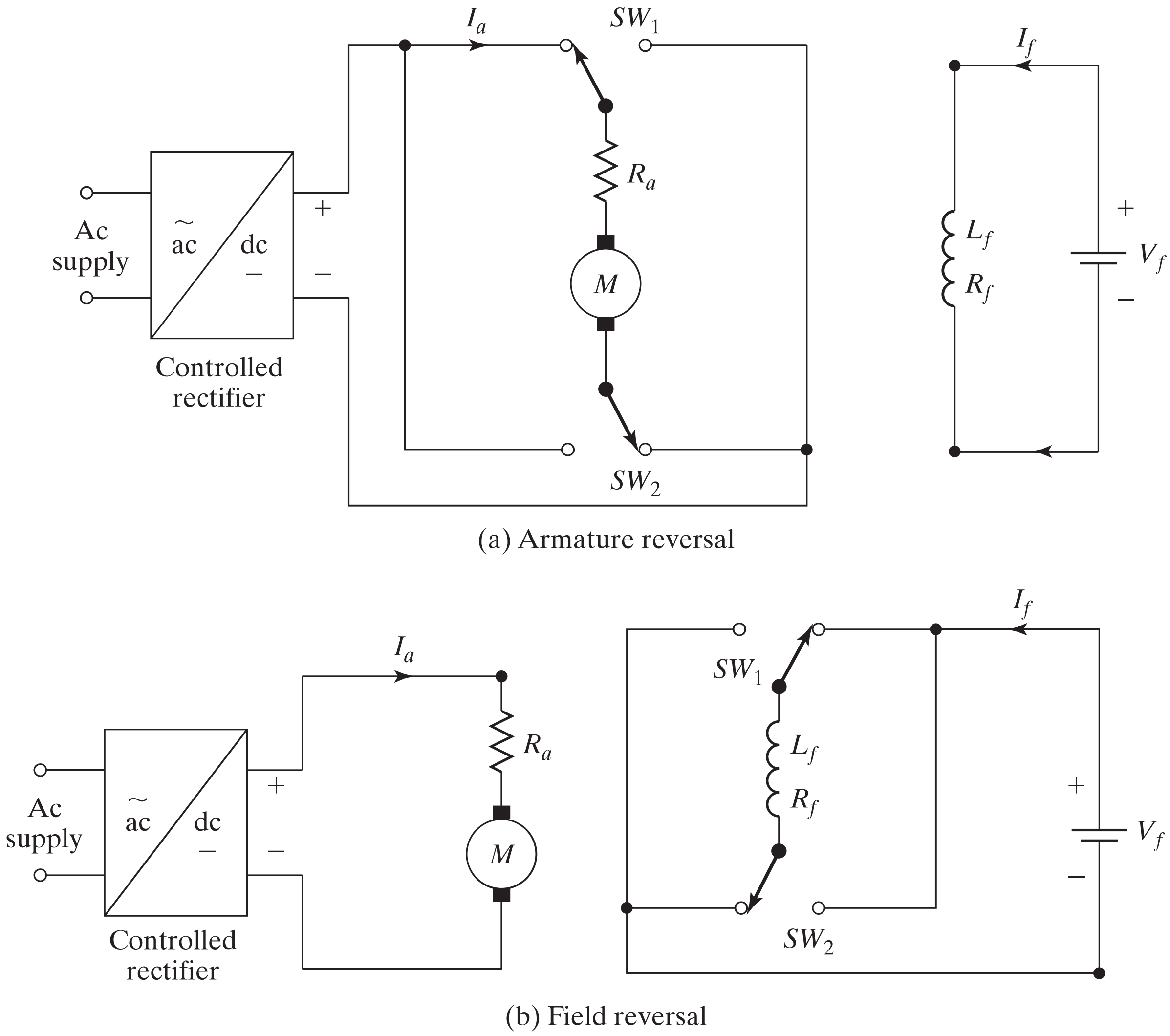


FIGURE 14.11

Field and armature reversals using contactors.

2 to 10 ms is provided to ensure that the armature current becomes zero. Because of a relatively large time constant of the field winding, the field reversal takes a longer time. A semi- or full converter can be used to vary the field voltage, but a full converter is preferable. Due to the ability to reverse the voltage, a full converter can reduce the field current much faster than a semiconverter. Depending on the type of single-phase converters, single-phase drives [4, 5] may be subdivided into:

1. Single-phase half-wave converter drives
2. Single-phase semiconverter drives
3. Single-phase full-converter drives
4. Single-phase dual-converter drives

The armature current of half-wave converter drives is normally discontinuous. This type of drive is not commonly used [12]. A semiconverter drive operates in one quadrant in applications up to 1.5 kW. The full converter and dual drives are more commonly used.

14.4.1 Single-Phase Semiconverter Drives

A single-phase semiconverter feeds the armature circuit, as shown in Figure 14.12a. It is a one-quadrant drive, as shown in Figure 14.12b, and is limited to applications up to 15 kW. The converter in the field circuit can be a semiconverter [12]. The current waveforms for a highly inductive load are shown in Figure 14.12c.

With a single-phase semiconverter in the armature circuit, the average armature voltage is can given by [12]

$$V_a = \frac{V_m}{\pi}(1 + \cos \alpha_a) \qquad \text{for } 0 \leq \alpha_a \leq \pi \tag{14.18}$$

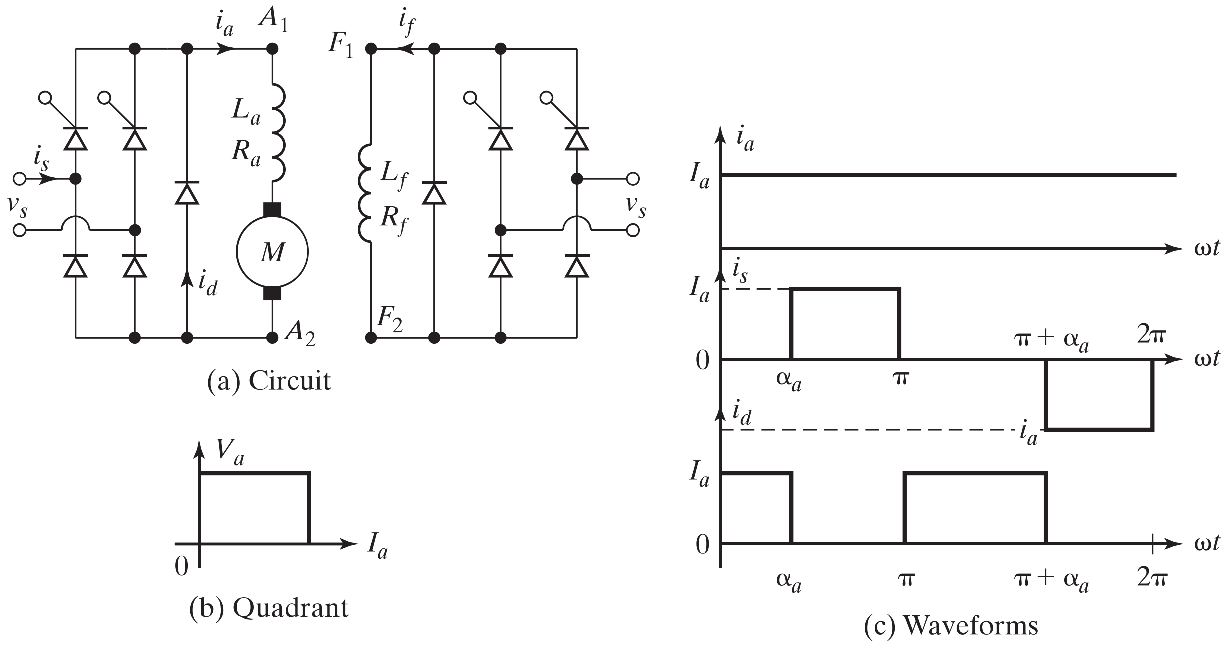


FIGURE 14.12

Single-phase semiconverter drive.

With a semiconverter in the field circuit, Eq. (10.52) gives the average field voltage as

$$V_f = \frac{V_m}{\pi}(1 + \cos\alpha_f) \qquad \text{for } 0 \leq \alpha_f \leq \pi \tag{14.19}$$

14.4.2 Single-Phase Full-Converter Drives

The armature voltage is varied by a single-phase full-wave converter, as shown in Figure 14.13a. It is a two-quadrant drive, as shown in Figure 14.13b, and is limited to applications up to 15 kW. The armature converter gives $+V_a$ or $-V_a$, and allows operation in the first and fourth quadrants. During regeneration for reversing the direction of power flow, the back emf of the motor can be reversed by reversing the field excitation. The converter in the field circuit could be a semi-, a full, or even a dual converter. The reversal of the armature or field allows operation in the second and third quadrants. The current waveforms for a highly inductive load are shown in Figure 14.13c for powering action.

With a single-phase full-wave converter in the armature circuit, Eq. (10.1) gives the average armature voltage as

$$V_a = \frac{2V_m}{\pi}\cos\alpha_a \qquad \text{for } 0 \leq \alpha_a \leq \pi \tag{14.20}$$

With a single-phase full-converter in the field circuit, Eq. (10.5) gives the field voltage as

$$V_f = \frac{2V_m}{\pi}\cos\alpha_f \qquad \text{for } 0 \leq \alpha_f \leq \pi \tag{14.21}$$

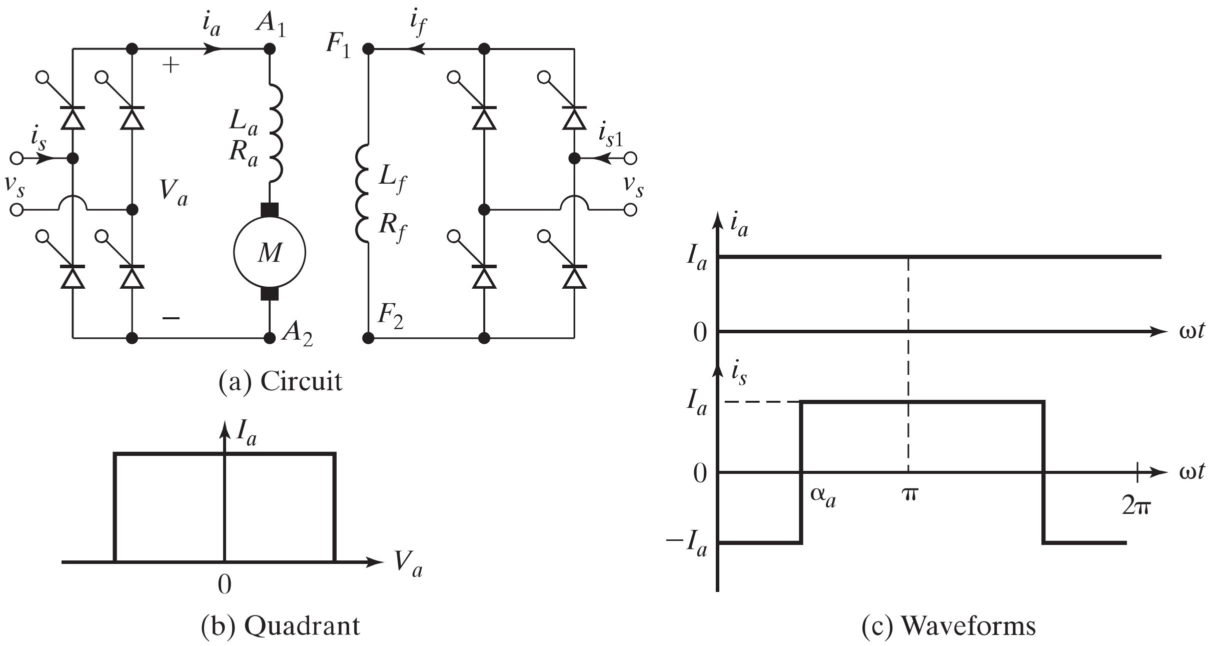


FIGURE 14.13

Single-phase full-converter drive.

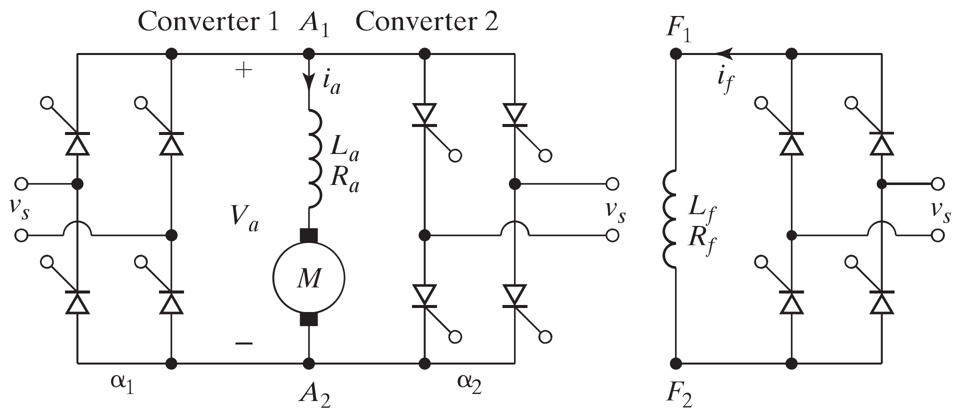


FIGURE 14.14

Single-phase dual-converter drive.

14.4.3 Single-Phase Dual-Converter Drives

Two single-phase full-wave converters are connected, as shown in Figure 14.14. Either converter 1 operates to supply a positive armature voltage, V_a, or converter 2 operates to supply a negative armature voltage, $-V_a$. Converter 1 provides operation in the first and fourth quadrants, and converter 2, in the second and third quadrants. It is a four-quadrant drive and permits four modes of operation: forward powering, forward braking (regeneration), reverse powering, and reverse braking (regeneration). It is limited to applications up to 15 kW. The field converter could be a full-wave, a semi-, or a dual converter.

If converter 1 operates with a delay angle of α_{a1}, Eq. (10.11) gives the armature voltage as

$$V_a = \frac{2V_m}{\pi}\cos\alpha_{a1} \qquad \text{for } 0 \le \alpha_{a1} \le \pi \tag{14.22}$$

If converter 2 operates with a delay angle of α_{a2}, Eq. (10.12) gives the armature voltage as

$$V_a = \frac{2V_m}{\pi}\cos\alpha_{a2} \qquad \text{for } 0 \le \alpha_{a2} \le \pi \tag{14.23}$$

where $\alpha_{a2} = \pi - \alpha_{a1}$. With a full converter in the field circuit, Eq. (10.1) gives the field voltage as

$$V_f = \frac{2V_m}{\pi}\cos\alpha_f \qquad \text{for } 0 \le \alpha_f \le \pi \tag{14.24}$$

Example 14.3 Finding the Performance Parameters of a Single-Phase Semiconverter Drive

The speed of a separately excited motor is controlled by a single-phase semiconverter in Figure 14.12a. The field current, which is also controlled by a semiconverter, is set to the maximum possible value. The ac supply voltage to the armature and field converters is one phase, 208 V, 60 Hz. The armature resistance is $R_a = 0.25\ \Omega$, the field resistance is $R_f = 147\ \Omega$, and the motor voltage constant is $K_v = 0.7032$ V/A rad/s. The load torque is $T_L = 45$ N·m at 1000 rpm.

The viscous friction and no-load losses are negligible. The inductances of the armature and field circuits are sufficient enough to make the armature and field currents continuous and ripple free. Determine (a) the field current I_f; (b) the delay angle of the converter in the armature circuit α_a, and (c) the input power factor of the armature circuit converter.

Solution

$V_s = 208$ V, $V_m = \sqrt{2} \times 208 = 294.16$ V, $R_a = 0.25\ \Omega$, $R_f = 147\ \Omega$, $T_d = T_L = 45$ N·m, $K_v = 0.7032$ V/A rad/s, and $\omega = 1000\ \pi/30 = 104.72$ rad/s.

a. From Eq. (14.19), the maximum field voltage (and current) is obtained for a delay angle of $\alpha_f = 0$ and

$$V_f = \frac{2V_m}{\pi} = \frac{2 \times 294.16}{\pi} = 187.27 \text{ V}$$

The field current is

$$I_f = \frac{V_f}{R_f} = \frac{187.27}{147} = 1.274 \text{ A}$$

b. From Eq. (14.4),

$$I_a = \frac{T_d}{K_v I_f} = \frac{45}{0.7032 \times 1.274} = 50.23 \text{ A}$$

From Eq. (14.2),

$$E_g = K_v \omega I_f = 0.7032 \times 104.72 \times 1.274 = 93.82 \text{ V}$$

From Eq. (14.3), the armature voltage is

$$V_a = 93.82 + I_a R_a = 93.82 + 50.23 \times 0.25 = 93.82 + 12.56 = 106.38 \text{ V}$$

From Eq. (14.18), $V_a = 106.38 = (294.16/\pi) \times (1 + \cos \alpha_a)$ and this gives the delay angle as $\alpha_a = 82.2°$.

c. If the armature current is constant and ripple free, the output power is $P_o = V_a I_a = 106.38 \times 50.23 = 5343.5$ W. If the losses in the armature converter are neglected, the power from the supply is $P_a = P_o = 5343.5$ W. The rms input current of the armature converter, as shown in Figure 14.12, is

$$I_{sa} = \left(\frac{2}{2\pi} \int_{\alpha_a}^{\pi} I_a^2 \, d\theta \right)^{1/2} = I_a \left(\frac{\pi - \alpha_a}{\pi} \right)^{1/2}$$

$$= 50.23 \left(\frac{180 - 82.2}{180} \right)^{1/2} = 37.03 \text{ A}$$

and the input volt–ampere (VA) rating is $\text{VI} = V_s I_{sa} = 208 \times 37.03 = 7702.24$. Assuming negligible harmonics, the input PF is approximately

$$\text{PF} = \frac{P_o}{\text{VI}} = \frac{5343.5}{7702.24} = 0.694 \text{ (lagging)}$$

$$PF = \frac{\sqrt{2}(1 + \cos 82.2°)}{[\pi(\pi - 82.2°)]^{1/2}} = 0.694 \text{ (lagging)}$$

The input power factor is given by [12]

$$PF = \frac{\sqrt{2}(1 + \cos\alpha)}{\sqrt{\pi(\pi + \cos\alpha)}}$$

Example 14.4 Finding the Performance Parameters of a Single-Phase Full-Converter Drive

The speed of a separately excited dc motor is controlled by a single-phase full-wave converter in Figure 14.13a. The field circuit is also controlled by a full converter and the field current is set to the maximum possible value. The ac supply voltage to the armature and field converters is one phase, 440 V, 60 Hz. The armature resistance is $R_a = 0.25\ \Omega$, the field circuit resistance is $R_f = 175\ \Omega$, and the motor voltage constant is $K_v = 1.4$ V/A rad/s. The armature current corresponding to the load demand is $I_a = 45$ A. The viscous friction and no-load losses are negligible. The inductances of the armature and field circuits are sufficient to make the armature and field currents continuous and ripple free. If the delay angle of the armature converter is $\alpha_a = 60°$ and the armature current is $I_a = 45$ A, determine (a) the torque developed by the motor T_d, (b) the speed ω, and (c) the input PF of the drive.

Solution

$V_s = 440$ V, $V_m = \sqrt{2} \times 440 = 622.25$ V, $R_a = 0.25\ \Omega$, $R_f = 175\ \Omega$, $\alpha_a = 60°$, and $K_v = 1.4$ V/A rad/s.

a. From Eq. (14.21), the maximum field voltage (and current) would be obtained for a delay angle of $\alpha_f = 0$ and

$$V_f = \frac{2V_m}{\pi} = \frac{2 \times 622.25}{\pi} = 396.14 \text{ V}$$

The field current is

$$I_f = \frac{V_f}{R_f} = \frac{396.14}{175} = 2.26 \text{ A}$$

From Eq. (14.4), the developed torque is

$$T_d = T_L = K_v I_f I_a = 1.4 \times 2.26 \times 45 = 142.4 \text{ N} \cdot \text{m}$$

From Eq. (14.20), the armature voltage is

$$V_a = \frac{2V_m}{\pi} \cos 60° = \frac{2 \times 622.25}{\pi} \cos 60° = 198.07 \text{ V}$$

The back emf is

$$E_g = V_a - I_a R_a = 198.07 - 45 \times 0.25 = 186.82 \text{ V}$$

b. From Eq. (14.2), the speed is

$$\omega = \frac{E_g}{K_v I_f} = \frac{186.82}{1.4 \times 2.26} = 59.05 \text{ rad/s or } 564 \text{ rpm}$$

c. Assuming lossless converters, the total input power from the supply is

$$P_i = V_a I_a + V_f I_f = 198.07 \times 45 + 396.14 \times 2.26 = 9808.4 \text{ W}$$

The input current of the armature converter for a highly inductive load is shown in Figure 14.13b and its rms value is $I_{sa} = I_a = 45$ A. The rms value of the input current of field converter is $I_{sf} = I_f = 2.26$ A. The effective rms supply current can be found from

$$I_s = (I_{sa}^2 + I_{sf}^2)^{1/2}$$

$$= (45^2 + 2.26^2)^{1/2} = 45.06 \text{ A}$$

and the input VA rating, $VI = V_s I_s = 440 \times 45.06 = 19{,}826.4$. Neglecting the ripples, the input power factor is approximately

$$\text{PF} = \frac{P_i}{\text{VI}} = \frac{9808.4}{19{,}826.4} = 0.495 \text{ (lagging)}$$

From Eq. (10.7),

$$\text{PF} = \left(\frac{2\sqrt{2}}{\pi}\right) \cos \alpha_a = \left(\frac{2\sqrt{2}}{\pi}\right) \cos 60° = 0.45 \text{ (lagging)}$$

Example 14.5 Finding the Delay Angle and Feedback Power in Regenerative Braking

If the polarity of the motor back emf in Example 14.4 is reversed by reversing the polarity of the field current, determine (a) the delay angle of the armature circuit converter, α_a, to maintain the armature current constant at the same value of $I_a = 45$ A; and (b) the power fed back to the supply due to regenerative braking of the motor.

Solution

a. From part (a) of Example 14.4, the back emf at the time of polarity reversal is $E_g = 186.82$ V and after polarity reversal $E_g = -186.82$ V. From Eq. (14.3),

$$V_a = E_g + I_a R_a = -186.82 + 45 \times 0.25 = -175.57 \text{ V}$$

From Eq. (14.20),

$$V_a = \frac{2V_m}{\pi} \cos \alpha_a = \frac{2 \times 622.25}{\pi} \cos \alpha_a = -175.57 \text{ V}$$

and this yields the delay angle of the armature converter as $\alpha_a = 116.31°$.

b. The power fed back to the supply is $P_a = V_a I_a = 175.57 \times 45 = 7900.7$ W.

Note: The speed and back emf of the motor decrease with time. If the armature current is to be maintained constant at $I_a = 45$ A during regeneration, the delay angle of the armature converter has to be reduced. This would require a closed-loop control to maintain the armature current constant and to adjust the delay angle continuously.

Key Points of Section 14.4

- A single-phase drive uses a single-phase converter. The type of single-phase converters classifies the single-phase drive.
- A semiconverter drive operates in one quadrant, a full-converter drive in two quadrants, and a dual converter in four quadrants. The field excitation is normally supplied from a full converter.

14.5 THREE-PHASE DRIVES

The armature circuit is connected to the output of a three-phase controlled rectifier or a forced-commutated three-phase ac–dc converter. Three-phase drives are used for high-power applications up to megawatt power levels. The ripple frequency of the armature voltage is higher than that of single-phase drives and it requires less inductance in the armature circuit to reduce the armature ripple current. The armature current is mostly continuous, and therefore the motor performance is better compared with that of single-phase drives. Similar to the single-phase drives, three-phase drives [2, 6] may also be subdivided into:

1. Three-phase half-wave-converter drives
2. Three-phase semiconverter drives
3. Three-phase full-converter drives
4. Three-phase dual-converter drives

The half-wave converters are not normally used in industrial applications and will not be covered further.

14.5.1 Three-Phase Semiconverter Drives

A three-phase semiconverter-fed drive is a one-quadrant drive without field reversal, and is limited to applications up to 115 kW. The field converter should also be a single-phase or a three-phase semiconverter.

With a three-phase semiconverter in the armature circuit, Eq. (10.69) gives the armature voltage as

$$V_a = \frac{3\sqrt{3}V_m}{2\pi}(1 + \cos \alpha_a) \qquad \text{for } 0 \le \alpha_a \le \pi \tag{14.25}$$

With a three-phase semiconverter in the field circuit, Eq. (10.69) gives the field voltage as

$$V_f = \frac{3\sqrt{3}V_m}{2\pi}(1 + \cos \alpha_f) \qquad \text{for } 0 \le \alpha_f \le \pi \tag{14.26}$$

14.5.2 Three-Phase Full-Converter Drives

A three-phase full-wave-converter drive is a two-quadrant drive without any field reversal, and is limited to applications up to 1500 kW. During regeneration for reversing the direction of power flow, the back emf of the motor is reversed by reversing the field excitation. The converter in the field circuit should be a single- or three-phase full converter.

With a three-phase full-wave converter in the armature circuit, Eq. (10.15) gives the armature voltage as

$$V_a = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha_a \qquad \text{for } 0 \leq \alpha_a \leq \pi \tag{14.27}$$

With a three-phase full converter in the field circuit, Eq. (10.15) gives the field voltage as

$$V_f = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha_f \qquad \text{for } 0 \leq \alpha_f \leq \pi \tag{14.28}$$

14.5.3 Three-Phase Dual-Converter Drives

Two three-phase full-wave converters are connected in an arrangement similar to Figure 14.15a. Either converter 1 operates to supply a positive armature voltage, V_a, or converter 2 operates to supply a negative armature voltage, $-V_a$. It is a four-quadrant drive and is limited to applications up to 1500 kW. Similar to single-phase drives, the field converter can be a full-wave converter or a semiconverter.

If converter 1 operates with a delay angle of α_{a1}, Eq. (10.15) gives the average armature voltage as

$$V_a = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha_{a1} \qquad \text{for } 0 \leq \alpha_{a1} \leq \pi \tag{14.29}$$

If converter 2 operates with a delay angle of α_{a2}, Eq. (10.15) gives the average armature voltage as

$$V_a = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha_{a2} \qquad \text{for } 0 \leq \alpha_{a2} \leq \pi \tag{14.30}$$

With a three-phase full converter in the field circuit, Eq. (10.15) gives the average field voltage as

$$V_f = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha_f \qquad \text{for } 0 \leq \alpha_f \leq \pi \tag{14.31}$$

Example 14.6 Finding the Performance Parameters of a Three-Phase Full-Converter Drive

The speed of a 20-hp, 300-V, 1800-rpm separately excited dc motor is controlled by a three-phase full-converter drive. The field current is also controlled by a three-phase full converter and is set to the maximum possible value. The ac input is a three-phase, Y-connected, 208-V, 60-Hz supply. The armature resistance is $R_a = 0.25\ \Omega$, the field resistance is $R_f = 245\ \Omega$, and the motor voltage constant is $K_v = 1.2$ V/A rad/s. The armature and field currents can be assumed to be continuous and ripple free. The viscous friction is negligible. Determine (a) the delay angle of the armature converter α_a, if the motor supplies the rated power at the rated speed; (b) the no-load speed if the delay angles are the same as in (a) and the armature current at no load is 10% of the rated value; and (c) the speed regulation.

Solution

$R_a = 0.25\ \Omega$, $R_f = 245\ \Omega$, $K_v = 1.2$ V/A rad/s, $V_L = 208$ V, and $\omega = 1800\ \pi/30 = 188.5$ rad/s. The phase voltage is $V_p = V_L/\sqrt{3} = 208/\sqrt{3} = 120$ V and $V_m = 120 \times \sqrt{2} = 169.7$ V. Because 1 hp is equal to 746 W, the rated armature current is $I_{rated} = 20 \times 746/300 = 49.73$ A; for maximum possible field current, $\alpha_f = 0$. From Eq. (14.28),

$$V_f = 3\sqrt{3} \times \frac{169.7}{\pi} = 280.7 \text{ V}$$

$$I_f = \frac{V_f}{R_f} = \frac{280.7}{245} = 1.146 \text{ A}$$

a. $I_a = I_{rated} = 49.73$ A and

$$E_g = K_v I_f \omega = 1.2 \times 1.146 \times 188.5 = 259.2 \text{ V}$$

$$V_a = 259.2 + I_a R_a = 259.2 + 49.73 \times 0.25 = 271.63 \text{ V}$$

From Eq. (14.27),

$$V_a = 271.63 = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha_a = \frac{3\sqrt{3} \times 169.7}{\pi}\cos\alpha_a$$

and this gives the delay angle as $\alpha_a = 14.59°$.

b. $I_a = 10\%$ of $49.73 = 4.973$ A and

$$E_{go} = V_a - R_a I_a = 271.63 - 0.25 \times 4.973 = 270.39 \text{ V}$$

From Eq. (14.4), the no-load speed is

$$\omega_0 = \frac{E_{go}}{K_v I_f} = \frac{270.39}{1.2 \times 1.146} = 196.62 \text{ rad/s} \quad \text{or} \quad 196.62 \times \frac{30}{\pi} = 1877.58 \text{ rpm}$$

c. The speed regulation is defined as

$$\frac{\text{no-load speed} - \text{full-load speed}}{\text{full-load speed}} = \frac{1877.58 - 1800}{1800} = 0.043 \quad \text{or} \quad 4.3\%$$

Example 14.7 Finding the Performance of a Three-Phase Full-Converter Drive with Field Control

The speed of a 20-hp, 300-V, 900-rpm separately excited dc motor is controlled by a three-phase full converter. The field circuit is also controlled by a three-phase full converter. The ac input to the armature and field converters is three-phase, Y-connected, 208 V, 60 Hz. The armature resistance is $R_a = 0.25\ \Omega$, the field circuit resistance is $R_f = 145\ \Omega$, and the motor voltage constant is $K_v = 1.2$ V/A rad/s. The viscous friction and no-load losses can be considered negligible. The armature and field currents are continuous and ripple free. (a) If the field converter is operated at the maximum field current and the developed torque is $T_d = 116$ N·m at 900 rpm, determine the delay angle of the armature converter α_a. (b) If the field circuit converter is set for the maximum field current, the developed torque is $T_d = 116$ N · m, and the delay angle of the armature converter is $\alpha_a = 0$, determine the speed of the motor. (c) For the same load demand as in (b), determine the delay angle of the field converter if the speed has to be increased to 1800 rpm.

Solution

$R_a = 0.25\ \Omega$, $R_f = 145\ \Omega$, $K_v = 1.2$ V/A rad/s, and $V_L = 208$ V. The phase voltage is $V_p = 208/\sqrt{3} = 120$ V and $V_m = \sqrt{2} \times 120 = 169.7$ V.

a. $T_d = 116$ N · m and $\omega = 900\ \pi/30 = 94.25$ rad/s. For maximum field current, $\alpha_f = 0$. From Eq. (14.28),

$$V_f = \frac{3 \times \sqrt{3} \times 169.7}{\pi} = 280.7 \text{ V}$$

$$I_f = \frac{280.7}{145} = 1.936 \text{ A}$$

From Eq. (14.4),

$$I_a = \frac{T_d}{K_v I_f} = \frac{116}{1.2 \times 1.936} = 49.93 \text{ A}$$

$$E_g = K_v I_f \omega = 1.2 \times 1.936 \times 94.25 = 218.96 \text{ V}$$

$$V_a = E_g + I_a R_a = 218.96 + 49.93 \times 0.25 = 231.44 \text{ V}$$

From Eq. (14.27),

$$V_a = 231.44 = \frac{3 \times \sqrt{3} \times 169.7}{\pi} \cos \alpha_a$$

which gives the delay angle as $\alpha_a = 34.46°$.

b. $\alpha_a = 0$ and

$$V_a = \frac{3 \times \sqrt{3} \times 169.7}{\pi} = 280.7 \text{ V}$$

$$E_g = 280.7 - 49.93 \times 0.25 = 268.22 \text{ V}$$

and the speed

$$\omega = \frac{E_g}{K_v I_f} = \frac{268.22}{1.2 \times 1.936} = 115.45 \text{ rad/s} \qquad \text{or} \qquad 1102.5 \text{ rpm}$$

c. $\omega = 1800\ \pi/30 = 188.5$ rad/s

$$E_g = 268.22 \text{ V} = 1.2 \times 188.5 \times I_f \qquad \text{or} \qquad I_f = 1.186 \text{ A}$$

$$V_f = 1.186 \times 145 = 171.97 \text{ V}$$

From Eq. (14.28),

$$V_f = 171.97 = \frac{3 \times \sqrt{3} \times 169.7}{\pi} \cos \alpha_f$$

which gives the delay angle as $\alpha_f = 52.2°$.

Key Points of Section 14.5

- A three-phase drive uses a three-phase converter. The type of three-phase converter classifies the three-phase drive. In general, a semiconverter drive operates in one quadrant; a full-converter drive, in two quadrants; and a dual-converter, in four quadrants. The field excitation is normally supplied from a full converter.

14.6 DC–DC CONVERTER DRIVES

Dc–dc converter (or simply chopper) drives are widely used in traction applications all over the world. A dc–dc converter is connected between a fixed-voltage dc source and a dc motor to vary the armature voltage. In addition to armature voltage control, a dc–dc converter can provide regenerative braking of the motors and can return energy back to the supply. This energy-saving feature is particularly attractive to transportation systems with frequent stops such as mass rapid transit (MRT). Dc–dc converter drives are also used in battery electric vehicles (BEVs). A dc motor can be operated in one of the four quadrants by controlling the armature or field voltages (or currents). It is often required to reverse the armature or field terminals to operate the motor in the desired quadrant.

If the supply is nonreceptive during the regenerative braking, the line voltage would increase and regenerative braking may not be possible. In this case, an alternative form of braking is necessary, such as rheostatic braking. The possible control modes of a dc–dc converter drive are:

1. Power (or acceleration) control
2. Regenerative brake control
3. Rheostatic brake control
4. Combined regenerative and rheostatic brake control

14.6.1 Principle of Power Control

The dc–dc converter is used to control the armature voltage of a dc motor. The circuit arrangement of a converter-fed dc separately excited motor is shown in Figure 14.15a. The dc–dc converter switch could be a transistor or an IGBT or a GTO dc–dc converter, as discussed in Section 5.3. This is a one-quadrant drive, as shown in Figure 14.15b. The waveforms for the armature voltage, load current, and input current are shown in Figure 14.15c, assuming a highly inductive load.

The average armature voltage is

$$V_a = kV_s \tag{14.32}$$

where k is the duty cycle of the dc–dc converter. The power supplied to the motor is

$$P_o = V_a I_a = kV_s I_a \tag{14.33}$$

where I_a is the average armature current of the motor and it is ripple free. Assuming a lossless dc–dc converter, the input power is $P_i = P_0 = kV_s I_s$. The average value of the input current is

$$I_s = kI_a \tag{14.34}$$

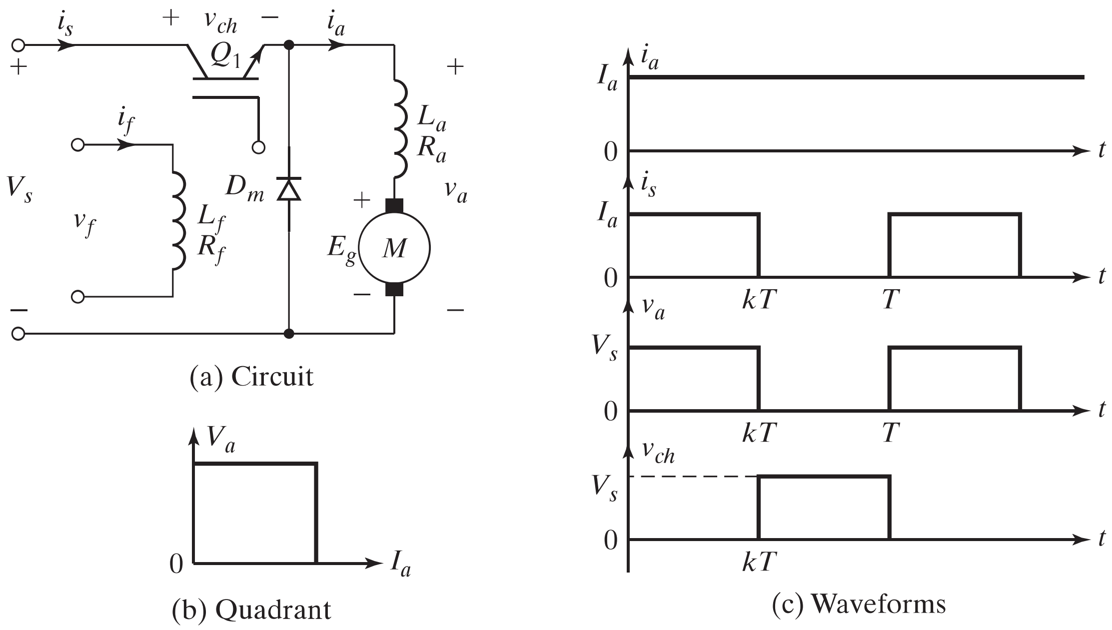


FIGURE 14.15

Converter-fed dc drive in power control.

The equivalent input resistance of the dc–dc converter drive seen by the source is

$$R_{eq} = \frac{V_s}{I_s} = \frac{V_s}{I_a}\frac{1}{k} \tag{14.35}$$

By varying the duty cycle k, the power flow to the motor (and speed) can be controlled. For a finite armature circuit inductance, Eq. (5.29) can be applied to find the maximum peak-to-peak ripple current as

$$\Delta I_{max} = \frac{V_s}{R_m} \tanh \frac{R_m}{4fL_m} \tag{14.36}$$

where R_m and L_m are the total armature circuit resistance and inductance, respectively. For a separately excited motor, $R_m = R_a +$ any series resistance, and $L_m = L_a +$ any series inductance. For a series motor, $R_m = R_a + R_f +$ any series resistance, and $L_m = L_a + L_f +$ any series inductance.

Example 14.8 Finding the Performance Parameters of a Dc–dc Converter Drive

A dc separately excited motor is powered by a dc–dc converter (as shown in Figure 14.15a), from a 600-V dc source. The armature resistance is $R_a = 0.05\ \Omega$. The back emf constant of the motor is $K_v = 1.527$ V/A rad/s. The average armature current is $I_a = 250$ A. The field current is $I_f = 2.5$ A. The armature current is continuous and has negligible ripple. If the duty cycle of the dc–dc converter is 60%, determine (a) the input power from the source, (b) the equivalent input resistance of the dc–dc converter drive, (c) the motor speed, and (d) the developed torque.

Solution

$V_s = 600$ V, $I_a = 250$ A, and $k = 0.6$. The total armature circuit resistance is $R_m = R_a = 0.05\ \Omega$.

a. From Eq. (14.33),

$$P_i = kV_sI_a = 0.6 \times 600 \times 250 = 90 \text{ kW}$$

b. From Eq. (14.35) $R_{eq} = 600/(250 \times 0.6) = 4\ \Omega$.

c. From Eq. (14.32), $V_a = 0.6 \times 600 = 360$ V. The back emf is

$$E_g = V_a - R_mI_m = 360 - 0.05 \times 250 = 347.5 \text{ V}$$

From Eq. (14.2), the motor speed is

$$\omega = \frac{347.5}{1.527 \times 2.5} = 91.03 \text{ rad/s} \quad \text{or} \quad 91.03 \times \frac{30}{\pi} = 869.3 \text{ rpm}$$

d. From Eq. (14.4),

$$T_d = 1.527 \times 250 \times 2.5 = 954.38 \text{ N}\cdot\text{m}$$

14.6.2 Principle of Regenerative Brake Control

In regenerative braking, the motor acts as a generator and the kinetic energy of the motor and load is returned back to the supply. The principle of energy transfer from one dc source to another of higher voltage is discussed in Section 5.4, and this can be applied in regenerative braking of dc motors.

The application of dc–dc converters in regenerative braking can be explained with Figure 14.16a. It requires rearranging the switch from powering mode to regenerative braking. Let us assume that the armature of a separately excited motor is rotating due to the inertia of the motor (and load), and in case of a transportation system, the kinetic energy of the vehicle or train would rotate the armature shaft. Then if the transistor is switched on, the armature current rises due to the short-circuiting of the motor terminals. If the dc–dc converter is turned off, diode D_m would be turned on and the energy stored in the armature circuit inductances would be transferred to the supply, provided that the supply is receptive. It is a one-quadrant drive and operates in the second quadrant, as shown in Figure 14.16b. Figure 14.16c shows the voltage and current waveforms assuming that the armature current is continuous and ripple free.

The average voltage across the dc–dc converter is

$$V_{ch} = (1 - k)V_s \tag{14.37}$$

If I_a is the average armature current, the regenerated power can be found from

$$P_g = I_aV_s(1 - k) \tag{14.38}$$

The voltage generated by the motor acting as a generator is

$$\begin{aligned} E_g &= K_vI_f\,\omega \\ &= V_{ch} + R_mI_a = (1 - k)V_s + R_mI_a \end{aligned} \tag{14.39}$$

where K_v is machine constant and ω is the machine speed in rads per second. Therefore, the equivalent load resistance of the motor acting as a generator is

$$R_{eq} = \frac{E_g}{I_a} = \frac{V_s}{I_a}(1 - k) + R_m \tag{14.40}$$

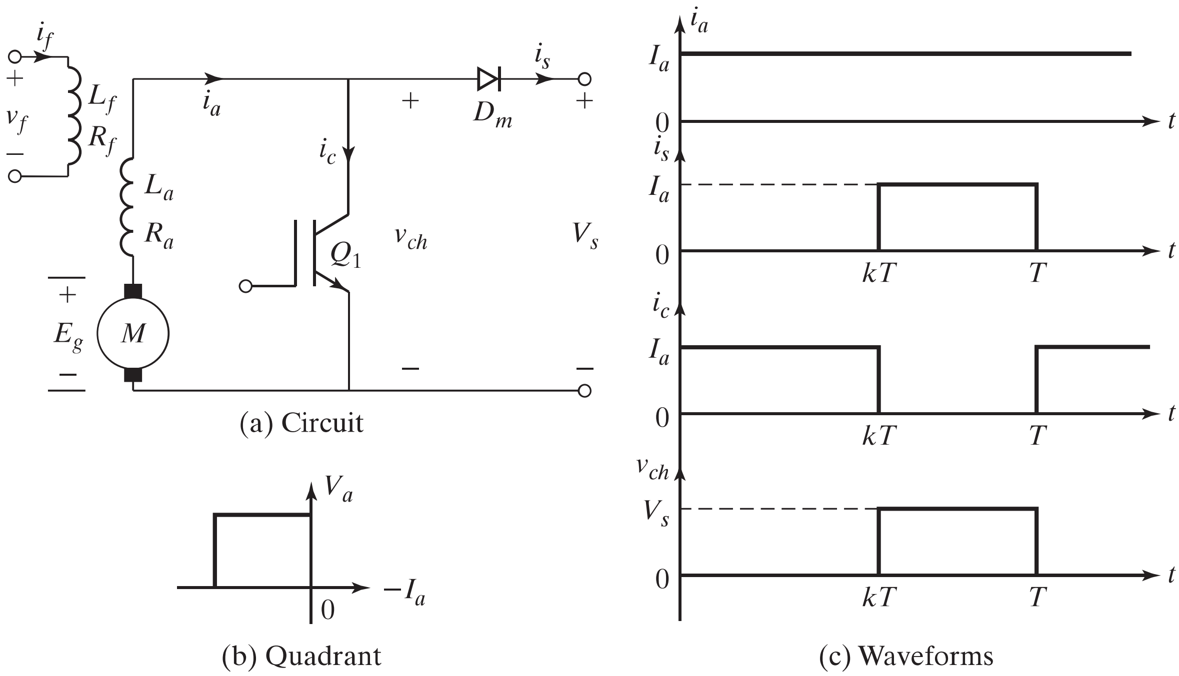


FIGURE 14.16

Regenerative braking of dc separately excited motors.

By varying the duty cycle k, the equivalent load resistance seen by the motor can be varied from R_m to $(V_s/I_a + R_m)$ and the regenerative power can be controlled.

From Eq. (5.38), the conditions for permissible potentials and polarity of the two voltages are

$$0 \leq (E_g - R_m I_a) \leq V_s \tag{14.41}$$

which gives the minimum braking speed of the motor as

$$E_g = K_v \omega_{\min} I_f = R_m I_a$$

or

$$\omega_{\min} = \frac{R_m}{K_v}\frac{I_a}{I_f} \tag{14.42}$$

and $\omega \geq \omega_{\min}$. The maximum braking speed of a series motor can be found from Eq. (14.41):

$$K_v \omega_{\max} I_f - R_m I_a = V_s$$

or

$$\omega_{\max} = \frac{V_s}{K_v I_f} + \frac{R_m}{K_v}\frac{I_a}{I_f} \tag{14.43}$$

and $\omega \leq \omega_{\max}$.

The regenerative braking would be effective only if the motor speed is between these two speed limits (e.g., $\omega_{\min} < \omega < \omega_{\max}$). At any speed less than $\omega_{\min}$, an alternative braking arrangement would be required.

Although dc series motors are traditionally used for traction applications due to their high-starting torque, a series-excited generator is unstable when working into a fixed voltage supply. Thus, for running on the traction supply, a separate excitation control is required and such an arrangement of series motor is, normally, sensitive to supply voltage fluctuations and a fast dynamic response is required to provide an adequate brake control. The application of a dc–dc converter allows the regenerative braking of dc series motors due to its fast dynamic response.

A separately excited dc motor is stable in regenerative braking. The armature and field can be controlled independently to provide the required torque during starting. A dc–dc converter-fed series and separately excited dc motors are both suitable for traction applications.

Example 14.9 Finding the Performance of a Dc–dc Converter-Fed Drive in Regenerative Braking

A dc–dc converter is used in regenerative braking of a dc series motor similar to the arrangement shown in Figure 14.16a. The dc supply voltage is 600 V. The armature resistance is $R_a = 0.02\ \Omega$ and the field resistance is $R_f = 0.03\ \Omega$. The back emf constant is $K_v = 15.27$ mV/A rad/s. The average armature current is maintained constant at $I_a = 250$ A. The armature current is continuous and has negligible ripple. If the duty cycle of the dc–dc converter is 60%, determine (a) the average voltage across the dc–dc converter V_{ch}; (b) the power regenerated to the dc supply P_g; (c) the equivalent load resistance of the motor acting as a generator, R_{eq}; (d) the minimum permissible braking speed ω_{min}; (e) the maximum permissible braking speed ω_{max}; and (f) the motor speed.

Solution

$V_s = 600$ V, $I_a = 250$ A, $K_v = 0.01527$ V/A rad/s, $k = 0.6$. For a series motor $R_m = R_a + R_f = 0.02 + 0.03 = 0.05\ \Omega$.

a. From Eq. (14.37), $V_{ch} = (1 - 0.6) \times 600 = 240$ V.

b. From Eq. (14.38), $P_g = 250 \times 600 \times (1 - 0.6) = 60$ kW.

c. From Eq. (14.40), $R_{eq} = (600/250)(1 - 0.6) + 0.05 = 1.01\ \Omega$.

d. From Eq. (14.42), the minimum permissible braking speed,

$$\omega_{min} = \frac{0.05}{0.01527} = 3.274 \text{ rad/s} \quad \text{or} \quad 3.274 \times \frac{30}{\pi} = 31.26 \text{ rpm}$$

e. From Eq. (14.43), the maximum permissible braking speed,

$$\omega_{max} = \frac{600}{0.01527 \times 250} + \frac{0.05}{0.01527} = 160.445 \text{ rad/s} \quad \text{or} \quad 1532.14 \text{ rpm}$$

f. From Eq. (14.39), $E_g = 240 + 0.05 \times 250 = 252.5$ V and the motor speed,

$$\omega = \frac{252.5}{0.01527 \times 250} = 66.14 \text{ rad/s or } 631.6 \text{ rpm}$$

Note: The motor speed would decrease with time. To maintain the armature current at the same level, the effective load resistance of the series generator should be adjusted by varying the duty cycle of the dc–dc converter.

14.6.3 Principle of Rheostatic Brake Control

In a rheostatic braking, the energy is dissipated in a rheostat and it may not be a desirable feature. In MRT systems, the energy may be used in heating the trains. The rheostatic braking is also known as *dynamic braking*. An arrangement for the rheostatic braking of a dc separately excited motor is shown in Figure 14.17a. This is a one-quadrant drive and operates in the second quadrant, as shown in Figure 14.17b. Figure 14.17c shows the waveforms for the current and voltage, assuming that the armature current is continuous and ripple free.

The average current of the braking resistor,

$$I_b = I_a(1 - k) \tag{14.44}$$

and the average voltage across the braking resistor,

$$V_b = R_b I_a(1 - k) \tag{14.45}$$

The equivalent load resistance of the generator,

$$R_{eq} = \frac{V_b}{I_a} = R_b(1 - k) + R_m \tag{14.46}$$

The power dissipated in the resistor R_b is

$$P_b = I_a^2 R_b(1 - k) \tag{14.47}$$

By controlling the duty cycle k, the effective load resistance can be varied from R_m to $R_m + R_b$, and the braking power can be controlled. The braking resistance R_b determines the maximum voltage rating of the dc–dc converter.

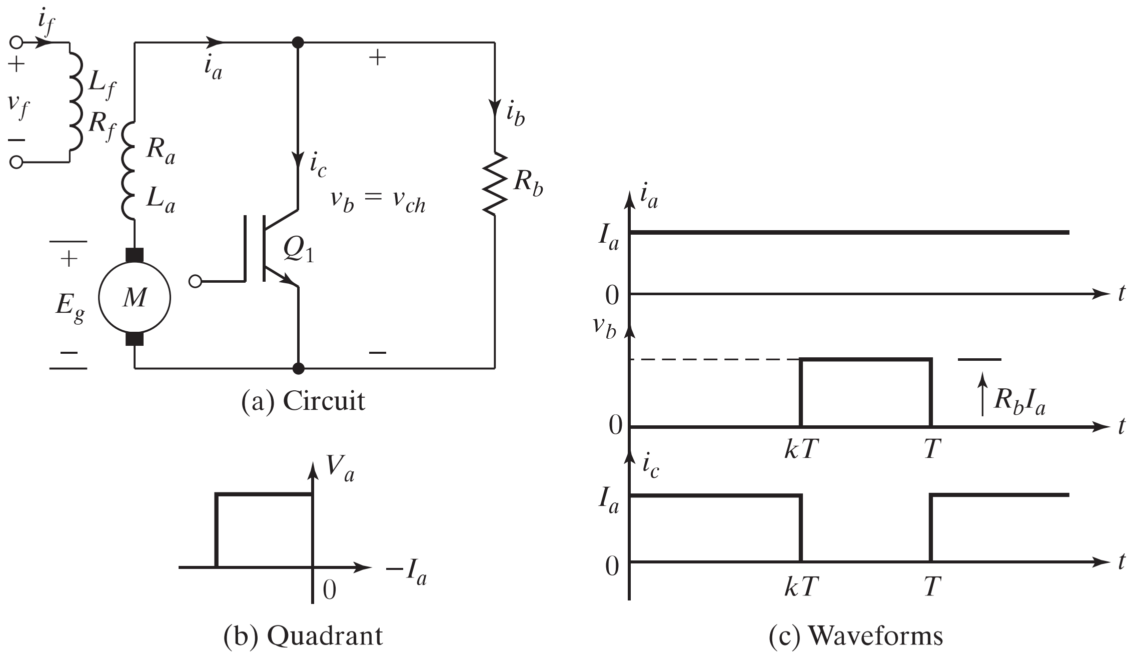


FIGURE 14.17

Rheostatic braking of dc separately excited motors.

Example 14.10 Finding the Performance of a Dc–dc Converter-Fed Drive in Rheostatic Braking

A dc–dc converter is used in rheostatic braking of a dc separately excited motor, as shown in Figure 14.17a. The armature resistance is $R_a = 0.05\ \Omega$. The braking resistor is $R_b = 5\ \Omega$. The back emf constant is $K_v = 1.527$ V/A rad/s. The average armature current is maintained constant at $I_a = 150$ A. The armature current is continuous and has negligible ripple. The field current is $I_f = 1.5$ A. If the duty cycle of the dc–dc converter is 40%, determine (a) the average voltage across the dc–dc converter V_{ch}; (b) the power dissipated in the braking resistor P_b; (c) the equivalent load resistance of the motor acting as a generator, R_{eq}; (d) the motor speed; and (e) the peak dc–dc converter voltage V_p.

Solution

$I_a = 150$ A, $K_v = 1.527$ V/A rad/s, $k = 0.4$, and $R_m = R_a = 0.05\ \Omega$.

a. From Eq. (14.45), $V_{ch} = V_b = 5 \times 150 \times (1 - 0.4) = 450$ V.

b. From Eq. (14.47), $P_b = 150 \times 150 \times 5 \times (1 - 0.4) = 67.5$ kW.

c. From Eq. (14.46), $R_{eq} = 5 \times (1 - 0.4) + 0.05 = 3.05\ \Omega$.

d. The generated emf $E_g = 450 + 0.05 \times 150 = 457.5$ V and the braking speed,

$$\omega = \frac{E_g}{K_v I_f} = \frac{457.5}{1.527 \times 1.5} = 199.74 \text{ rad/s} \qquad \text{or} \qquad 1907.4 \text{ rpm}$$

e. The peak dc–dc converter voltage $V_p = I_a R_b = 150 \times 5 = 750$ V.

14.6.4 Principle of Combined Regenerative and Rheostatic Brake Control

Regenerative braking is energy-efficient braking. On the other hand, the energy is dissipated as heat in rheostatic braking. If the supply is partly receptive, which is normally the case in practical traction systems, a combined regenerative and rheostatic brake control would be the most energy efficient. Figure 14.18 shows an arrangement in which rheostatic braking is combined with regenerative braking.

During regenerative brakings, the line voltage is sensed continuously. If it exceeds a certain preset value, normally 20% above the line voltage, the regenerative braking is removed and a rheostatic braking is applied. It allows an almost instantaneous

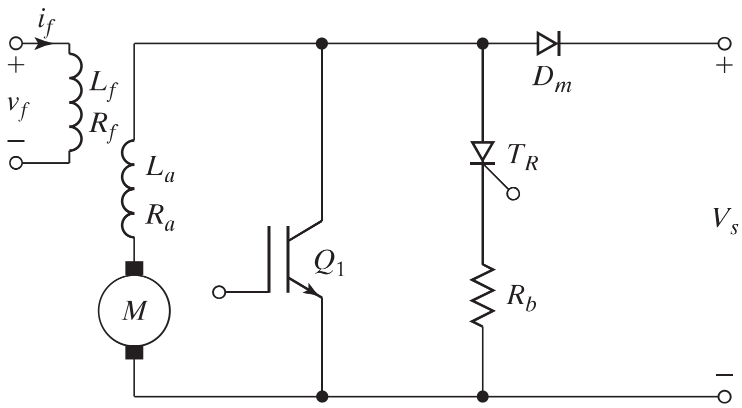


FIGURE 14.18

Combined regenerative and rheostatic braking.

transfer from regenerative to rheostatic braking if the line becomes nonreceptive, even momentarily. In every cycle, the logic circuit determines the receptivity of the supply. If it is nonreceptive, thyristor T_R is turned on to divert the motor current to the resistor R_b. Thyristor T_R is self-commutated when transistor Q_1 is turned on in the next cycle.

14.6.5 Two- and Four-Quadrant Dc–dc Converter Drives

During power control, a dc–dc converter-fed drive operates in the first quadrant, where the armature voltage and armature current are positive, as shown in Figure 14.15b. In a regenerative braking, the dc–dc converter drive operates in the second quadrant, where the armature voltage is positive and the armature current is negative, as shown in Figure 14.16b. Two-quadrant operation, as shown in Figure 14.19a, is required to allow power and regenerative braking control. The circuit arrangement of a transistorized two-quadrant drive is shown in Figure 14.19b.

Power control. Transistor Q_1 and diode D_2 operate. When Q_1 is turned on, the supply voltage V_s is connected to the motor terminals. When Q_1 is turned off, the armature current that flows through the freewheeling diode D_2 decays.

Regenerative control. Transistor Q_2 and diode D_1 operate. When Q_2 is turned on, the motor acts as a generator and the armature current rises. When Q_2 is turned off, the motor, acting as a generator, returns energy to the supply through the regenerative diode D_1. In industrial applications, four-quadrant operation, as shown in Figure 14.20a, is required. A transistorized four-quadrant drive is shown in Figure 14.20b.

Forward power control. Transistors Q_1 and Q_2 operate. Transistors Q_3 and Q_4 are off. When Q_1 and Q_2 are turned on together, the supply voltage appears across the motor terminals and the armature current rises. When Q_1 is turned off and Q_2 is still

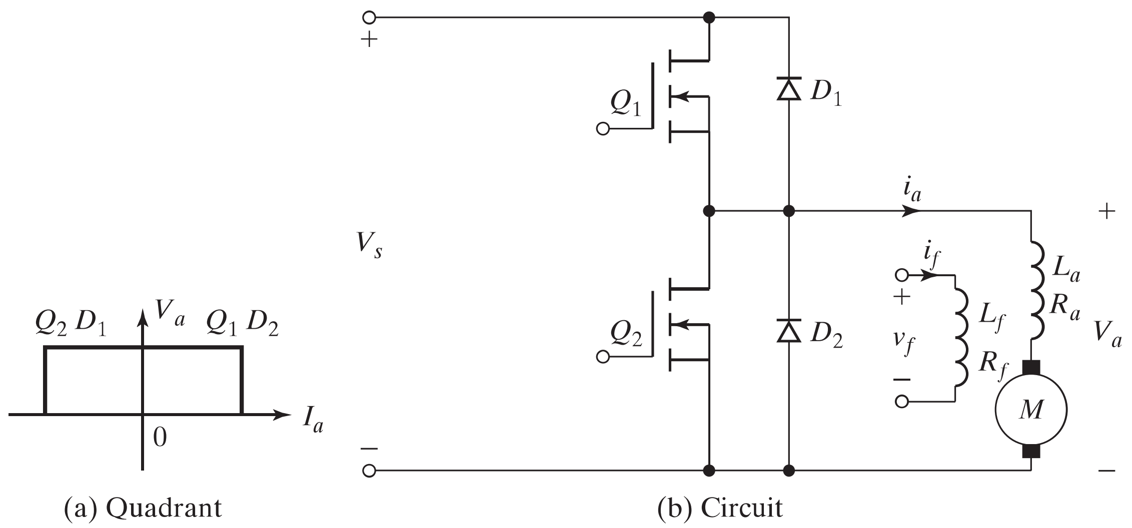


FIGURE 14.19

Two-quadrant transistorized dc–dc converter drive.

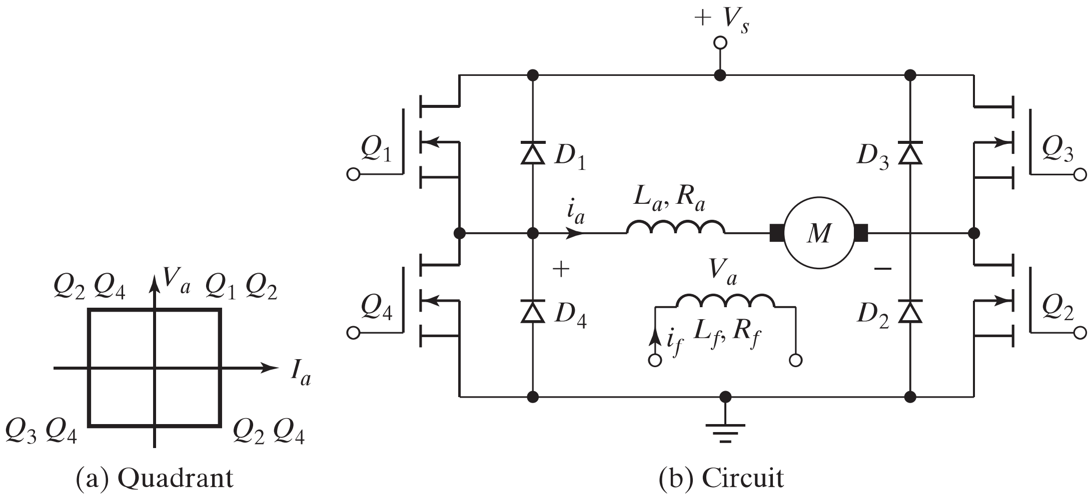


FIGURE 14.20

Four-quadrant transistorized dc—dc converter drive.

turned on, the armature current decays through Q_2 and D_4. Alternatively, both Q_1 and Q_2 can be turned off, while the armature current is forced to decay through D_3 and D_4.

Forward regeneration. Transistors Q_1, Q_2, and Q_3 are turned off. When transistor Q_4 is turned on, the armature current, which rises, flows through Q_4 and D_2. When Q_4 is turned off, the motor, acting as a generator, returns energy to the supply through D_1 and D_2.

Reverse power control. Transistors Q_3 and Q_4 operate. Transistors Q_1 and Q_2 are off. When Q_3 and Q_4 are turned on together, the armature current rises and flows in the reverse direction. When Q_3 is turned off and Q_4 is turned on, the armature current falls through Q_4 and D_2. Alternatively, both Q_3 and Q_4 can be turned off, while forcing the armature current to decay through D_1 and D_2.

Reverse regeneration. Transistors Q_1, Q_3, and Q_4 are off. When Q_2 is turned on, the armature current rises through Q_2 and D_4. When Q_2 is turned off, the armature current falls and the motor returns energy to the supply through D_3 and D_4.

14.6.6 Multiphase Dc–dc Converters

If two or more dc–dc converters are operated in parallel and are phase shifted from each other by π/u, as shown in Figure 14.21a, the amplitude of the load current ripples decreases and the ripple frequency increases [7, 8]. As a result, the dc–dc converter-generated harmonic currents in the supply are reduced. The sizes of the input filter are also reduced. The multiphase operation allows the reduction of smoothing chokes, which are normally connected in the armature circuit of dc motors. Separate inductors in each phase are used for current sharing. Figure 14.21b shows the waveforms for the currents with u dc–dc converters.

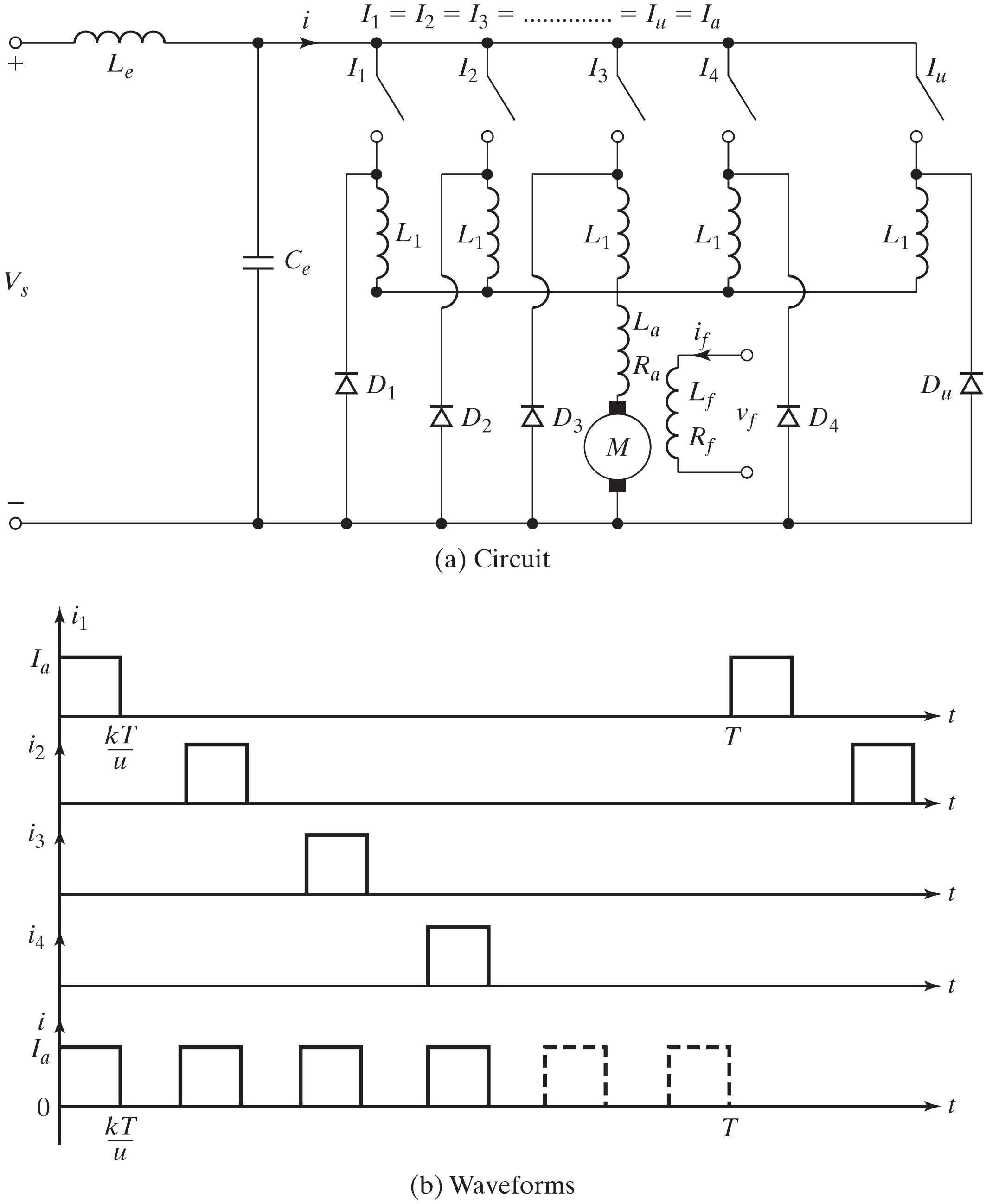


FIGURE 14.21

Multiphase dc–dc converters.

For u dc–dc converters in multiphase operation, it can be proved that Eq. (5.29) is satisfied when $k = 1/2u$ and the maximum peak-to-peak load ripple current becomes

$$\Delta I_{\max} = \frac{V_s}{R_m} \tanh \frac{R_m}{4ufL_m} \tag{14.48}$$

where L_m and R_m are the total armature inductance and resistance, respectively. For $4ufL_m \gg R_m$, the maximum peak-to-peak load ripple current can be approximated to

$$\Delta I_{\max} = \frac{V_s}{4ufL_m} \tag{14.49}$$

If an LC-input filter is used, Eq. (5.125) can be applied to find the rms nth harmonic component of dc–dc converter-generated harmonics in the supply

$$I_{ns} = \frac{1}{1 + (2n\pi uf)^2 L_e C_e} I_{nh}$$

$$= \frac{1}{1 + (nuf/f_0)^2} I_{nh} \tag{14.50}$$

where I_{nh} is the rms value of the nth harmonic component of the dc–dc converter current and $f_0 = [1/(2\pi L_e C_e)]$ is the resonant frequency of the input filter. If $(nuf/f_0) \gg 1$, the nth harmonic current in the supply becomes

$$I_{ns} = I_{nh}\left(\frac{f_0}{nuf}\right)^2 \tag{14.51}$$

Multiphase operations are advantageous for large motor drives, especially if the load current requirement is large. However, considering the additional complexity involved in increasing the number of dc–dc converters, there is not much reduction in the dc–dc converter-generated harmonics in the supply line if more than two dc–dc converters are used [7]. In practice, both the magnitude and frequency of the line current harmonics are important factors to determine the level of interferences into the signaling circuits. In many rapid transit systems, the power and signaling lines are very close; in three-line systems, they even share a common line. The signaling circuits are sensitive to particular frequencies and the reduction in the magnitude of harmonics by using a multiphase operation of dc–dc converters could generate frequencies within the sensitivity band—which might cause more problems than it solves.

Example 14.11 Finding the Peak Load Current Ripple of Two Multiphase Dc–dc Converters

Two dc–dc converters control a dc separately excited motor and they are phase shifted in operation by $\pi/2$. The supply voltage to the dc–dc converter drive is $V_s = 220$ V, the total armature circuit resistance is $R_m = 4\ \Omega$, the total armature circuit inductance is $L_m = 15$ mH, and the frequency of each dc–dc converter is $f = 350$ Hz. Calculate the maximum peak-to-peak load ripple current.

Solution

The effective chopping frequency is $f_e = 2 \times 350 = 700$ Hz, $R_m = 4\ \Omega$, $L_m = 15$ mH, $u = 2$, and $V_s = 220$ V. $4ufL_m = 4 \times 2 \times 350 \times 15 \times 10^{-3} = 42$. Because $42 \gg 4$, approximate Eq. (14.49) can be used, and this gives the maximum peak-to-peak load ripple current, $\Delta I_{max} = 220/42 = 5.24$ A.

Example 14.12 Finding the Line Harmonic Current of Two Multiphase Dc–dc Converters with an Input Filter

A dc separately excited motor is controlled by two multiphase dc–dc converters. The average armature current is $I_a = 100$ A. A simple LC-input filter with $L_e = 0.3$ mH and $C_e = 4500\ \mu$F is used. Each dc–dc converter is operated at a frequency of $f = 350$ Hz. Determine the rms fundamental component of the dc–dc converter-generated harmonic current in the supply.

Solution

$I_a = 100$ A, $u = 2$, $L_e = 0.3$ mH, $C_e = 4500$ μF, and $f_0 = 1/(2\pi\sqrt{L_e C_e}) = 136.98$ Hz. The effective dc–dc converter frequency is $f_e = 2 \times 350 = 700$ Hz. From the results of Example 5.9, the rms value of the fundamental component of the dc–dc converter current is $I_{1h} = 45.02$ A. From Eq. (14.50), the fundamental component of the dc–dc converter-generated harmonic current is

$$I_{1s} = \frac{45.02}{1 + (2 \times 350/136.98)^2} = 1.66 \text{ A}$$

Key Points of Section 14.6

- It is possible to arrange the dc–dc converter in powering the motor, or in regenerative or rheostatic braking. A dc–dc converter-fed drive can operate in four quadrants.
- Multiphase dc–dc converters are often used to supply load current that cannot be handled by one dc–dc converter. Multiphase dc–dc converters have the advantage of increasing the effective converter frequency, thereby reducing the component values and size of the input filter.

14.7 CLOSED-LOOP CONTROL OF DC DRIVES

The speed of dc motors changes with the load torque. To maintain a constant speed, the armature (and or field) voltage should be varied continuously by varying the delay angle of ac–dc converters or duty cycle of dc–dc converters. In practical drive systems it is required to operate the drive at a constant torque or constant power; in addition, controlled acceleration and deceleration are required. Most industrial drives operate as closed-loop feedback systems. A closed-loop control system has the advantages of improved accuracy, fast dynamic response, and reduced effects of load disturbances and system nonlinearities [9].

The block diagram of a closed-loop converter-fed separately excited dc drive is shown in Figure 14.22. If the speed of the motor decreases due to the application of additional load torque, the speed error V_e increases. The speed controller responses with an increased control signal V_c change the delay angle or duty cycle of the converter, and increase the armature voltage of the motor. An increased armature voltage develops more torque to restore the motor speed to the original value. The drive normally passes through a transient period until the developed torque is equal to the load torque.

14.7.1 Open-Loop Transfer Function

The steady-state characteristics of dc drives, which are discussed in the preceding sections, are of major importance in the selection of dc drives and are not sufficient when the drive is in closed-loop control. Knowledge of the dynamic behavior, which is normally expressed in the form of a transfer function, is also important.

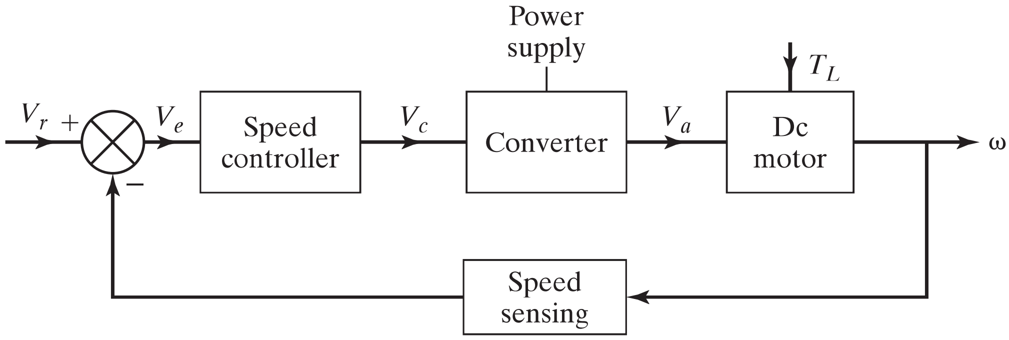


FIGURE 14.22

Block diagram of a closed-loop converter-fed dc motor drive.

14.7.2 Open-Loop Transfer Function of Separately Excited Motors

The circuit arrangement of a converter-fed separately excited dc motor drive with open-loop control is shown in Figure 14.23. The motor speed is adjusted by setting reference (or control) voltage v_r. Assuming a linear power converter of gain K_2, the armature voltage of the motor is

$$v_a = K_2 v_r \tag{14.52}$$

Assuming that the motor field current I_f and the back emf constant K_v remain constant during any transient disturbances, the system equations are

$$e_g = K_v I_f \omega \tag{14.53}$$

$$v_a = R_m i_a + L_m \frac{di_a}{dt} + e_g = R_m i_a + L_m \frac{di_a}{dt} + K_v I_f \omega \tag{14.54}$$

$$T_d = K_t I_f i_a \tag{14.55}$$

$$T_d = K_t I_f i_a = J \frac{d\omega}{dt} + B\omega + T_L \tag{14.56}$$

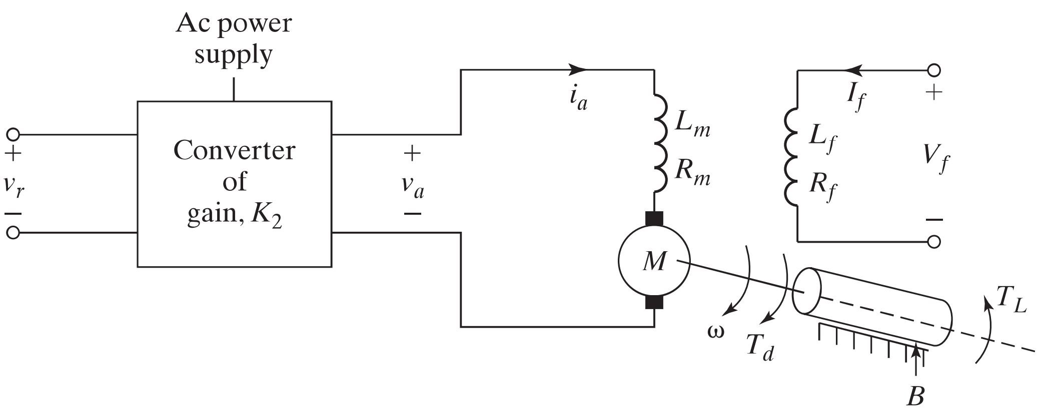


FIGURE 14.23

Converter-fed separately excited dc motor drive.

The dynamic characteristics of the motor described by Eqs (14.54) to (14.56) can be represented in the state-space form

$$\begin{bmatrix} pi_a \\ p\omega_m \end{bmatrix} = \begin{pmatrix} -\frac{R_m}{L_m} & -\frac{K_b}{L_m} \\ \frac{K_b}{J} & -\frac{B}{J} \end{pmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} + \begin{pmatrix} \frac{1}{L_m} & 0 \\ 0 & -\frac{1}{J} \end{pmatrix} \begin{bmatrix} v_a \\ T_L \end{bmatrix} \tag{14.57}$$

where p is the differential operator with respect to time and $K_b = K_v I_f$ is a back emf constant. Equation (14.57) can be expressed in generalized state-space form

$$\dot{X} = AX + BU \tag{14.58}$$

where $X = [i_a \ \ \omega_m]^T$ is the state variable vector and $U = [v_a \ \ T_L]^T$ is the input vector.

$$A = \begin{pmatrix} -\frac{R_m}{L_m} & -\frac{K_b}{L_m} \\ \frac{K_b}{J} & -\frac{B}{J} \end{pmatrix} \quad B = \begin{pmatrix} \frac{1}{L_m} & 0 \\ 0 & -\frac{1}{J} \end{pmatrix} \tag{14.59}$$

The roots of the quadratic system can be determined from the A matrix as given by

$$r_1; r_2 = \frac{-\left(\frac{R_m}{L_m} + \frac{B}{J}\right) \pm \sqrt{\left(\frac{R_m}{L_m} + \frac{B}{J}\right)^2 - 4\left(\frac{R_m B + K_b^2}{J L_m}\right)}}{2} \tag{14.60}$$

It should be noted that the roots of the system will always be negative real. That is, the motor is stable in an open-loop operation.

The transient behavior may be analyzed by changing the system equations into the Laplace transforms with zero initial conditions. Transforming Eqs. (14.52), (14.54), and (14.56) yields

$$V_a(s) = K_2 V_r(s) \tag{14.61}$$

$$V_a(s) = R_m I_a(s) + sL_m I_a(s) + K_v I_f \omega(s) \tag{14.62}$$

$$T_d(s) = K_t I_f I_a(s) = sJ\omega(s) + B\omega(s) + T_L(s) \tag{14.63}$$

From Eq. (14.62), the armature current is

$$I_a(s) = \frac{V_a(s) - K_v I_f \omega(s)}{sL_m + R_m} \tag{14.64}$$

$$= \frac{V_a(s) - K_v I_f \omega(s)}{R_m(s\tau_a + 1)} \tag{14.65}$$

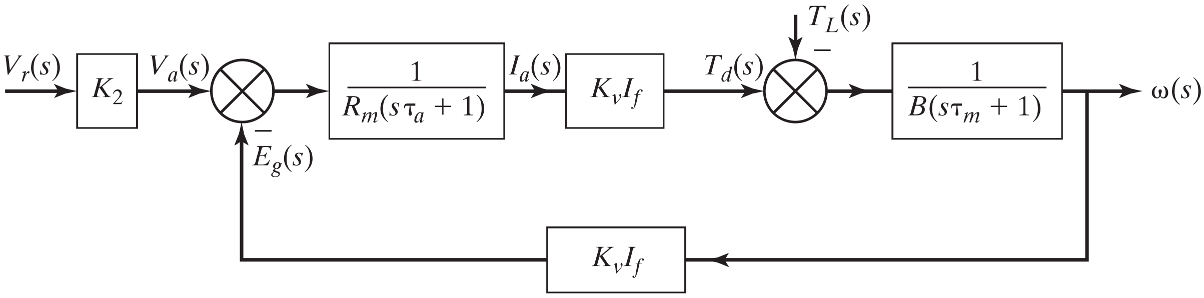


FIGURE 14.24

Open-loop block diagram of separately excited dc motor drive.

where $\tau_a = L_m/R_m$ is known as the *time constant* of motor armature circuit. From Eq. (14.63), the motor speed is

$$\omega(s) = \frac{T_d(s) - T_L(s)}{sJ + B} \tag{14.66}$$

$$= \frac{T_d(s) - T_L(s)}{B(s\tau_m + 1)} \tag{14.67}$$

where $\tau_m = J/B$ is known as the *mechanical time constant* of the motor. Equations (14.61), (14.65), and (14.67) can be used to draw the open-loop block diagram as shown in Figure 14.24. Two possible disturbances are control voltage V_r and load torque T_L. The steady-state responses can be determined by combining the individual response due to V_r and T_L.

The response due to a step change in the reference voltage is obtained by setting T_L to zero. From Figure 14.24, we can obtain the speed response due to reference voltage as

$$\frac{\omega(s)}{V_r(s)} = G_{\omega-V} = \frac{K_2 K_v I_f/(R_m B)}{s^2(\tau_a\tau_m) + s(\tau_a + \tau_m) + 1 + (K_v I_f)^2/R_m B} \tag{14.68}$$

The response due to a change in load torque T_L can be obtained by setting V_r to zero. The block diagram for a step change in load torque disturbance is shown in Figure 14.25.

$$\frac{\omega(s)}{T_L(s)} = G_{\omega-T} = -\frac{(1/B)(s\tau_a + 1)}{s^2(\tau_a\tau_m) + s(\tau_a + \tau_m) + 1 + (K_v I_f)^2/R_m B} \tag{14.69}$$

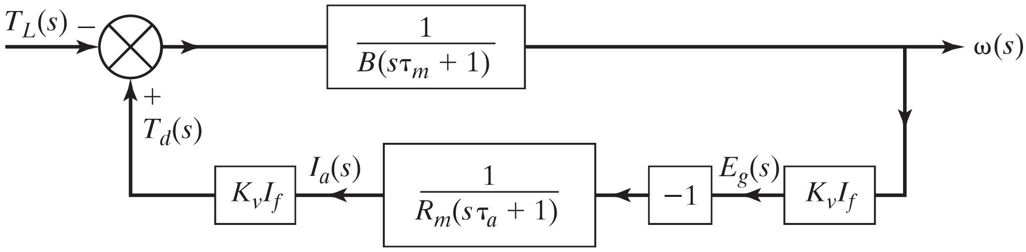


FIGURE 14.25

Open-loop block diagram for torque disturbance input.

Using the final value theorem, the steady-state relationship of a change in speed $\Delta\omega$, due to a step change in control voltage ΔV_r, and a step change in load torque ΔT_L, can be found from Eqs. (14.68) and (14.69), respectively, by substituting $s = 0$.

$$\Delta\omega = \frac{K_2 K_v I_f}{R_m B + (K_v I_f)^2} \Delta V_r \tag{14.70}$$

$$\Delta\omega = -\frac{R_m}{R_m B + (K_v I_f)^2} \Delta T_L \tag{14.71}$$

The speed response due to simultaneous applications of disturbances in both input reference voltage V_r and the load torque T_L can be found by summing their individual responses. Eqs (14.68) and (14.69) give the speed response as

$$\omega(s) = G_{\omega-V} V_r + G_{\omega-T} T_L \tag{14.72}$$

14.7.3 Open-Loop Transfer Function of Series Excited Motors

The dc series motors are used extensively in traction applications where the steady-state speed is determined by the friction and gradient forces. By adjusting the armature voltage, the motor may be operated at a constant torque (or current) up to the base speed, which corresponds to the maximum armature voltage. A dc–dc converter-controlled dc series motor drive is shown in Figure 14.26.

The armature voltage is related to the control (or reference) voltage by a linear gain of dc–dc converter K_2. Assuming that the back emf constant K_v does not change with the armature current and remains constant, the system equations are

$$v_a = K_2 v_r \tag{14.73}$$

$$e_g = K_v i_a \omega \tag{14.74}$$

$$v_a = R_m i_a + L_m \frac{di_a}{dt} + e_g \tag{14.75}$$

$$T_d = K_t i_a^2 \tag{14.76}$$

$$T_d = J\frac{d\omega}{dt} + B\omega + T_L \tag{14.77}$$

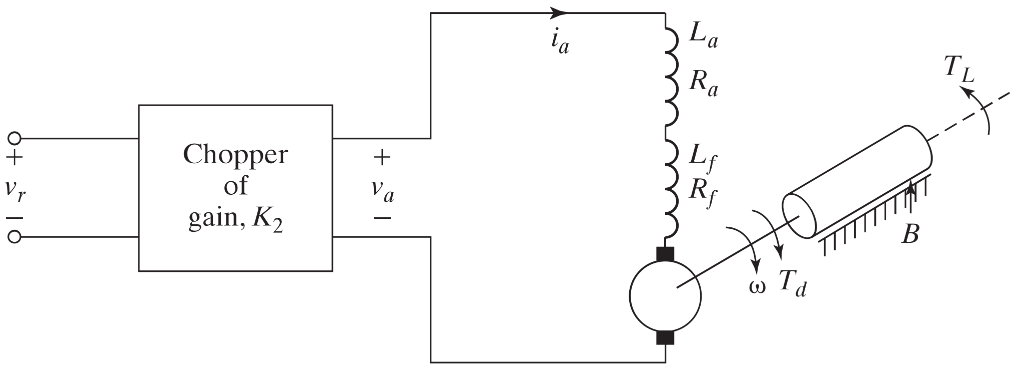


FIGURE 14.26

Dc–dc converter-fed dc series motor drive.

Equation (14.76) contains a product of variable-type nonlinearities, and as a result the application of transfer function techniques would no longer be valid. However, these equations can be linearized by considering a small perturbation at the operating point.

Let us define the system parameters around the operating point as

$$e_g = E_{g0} + \Delta e_g \quad i_a = I_{a0} + \Delta i_a \quad v_a = V_{a0} + \Delta v_a \quad T_d = T_{d0} + \Delta T_d$$
$$\omega = \omega_0 + \Delta\omega \quad v_r = V_{r0} + \Delta v_r \quad T_L = T_{L0} + \Delta T_L$$

Recognizing that $\Delta i_a \, \Delta\omega$ and $(\Delta i_a)^2$ are very small, tending to zero, Eqs. (14.73) to (14.77) can be linearized to

$$\Delta v_a = K_2 \, \Delta v_r$$
$$\Delta e_g = K_v (I_{a0} \, \Delta\omega + \omega_0 \, \Delta i_a)$$
$$\Delta v_a = R_m \, \Delta i_a + L_m \frac{d(\Delta i_a)}{dt} + \Delta e_g$$
$$\Delta T_d = 2K_v I_{a0} \, \Delta i_a$$
$$\Delta T_d = J \frac{d(\Delta\omega)}{dt} + B \, \Delta\omega + \Delta T_L$$

Transforming these equations in the Laplace domain gives us

$$\Delta V_a(s) = K_2 \, \Delta V_r(s) \tag{14.78}$$
$$\Delta E_g(s) = K_v [I_{a0} \, \Delta\omega(s) + \omega_0 \, \Delta I_a(s)] \tag{14.79}$$
$$\Delta V_a(s) = R_m \, \Delta I_a(s) + sL_m \, \Delta I_a(s) + \Delta E_g(s) \tag{14.80}$$
$$\Delta T_d(s) = 2K_v I_{a0} \, \Delta I_a(s) \tag{14.81}$$
$$\Delta T_d(s) = sJ \, \Delta\omega(s) + B \, \Delta\omega(s) + \Delta T_L(s) \tag{14.82}$$

These five equations are sufficient to establish the block diagram of a dc series motor drive, as shown in Figure 14.27. It is evident from Figure 14.27 that any change in either

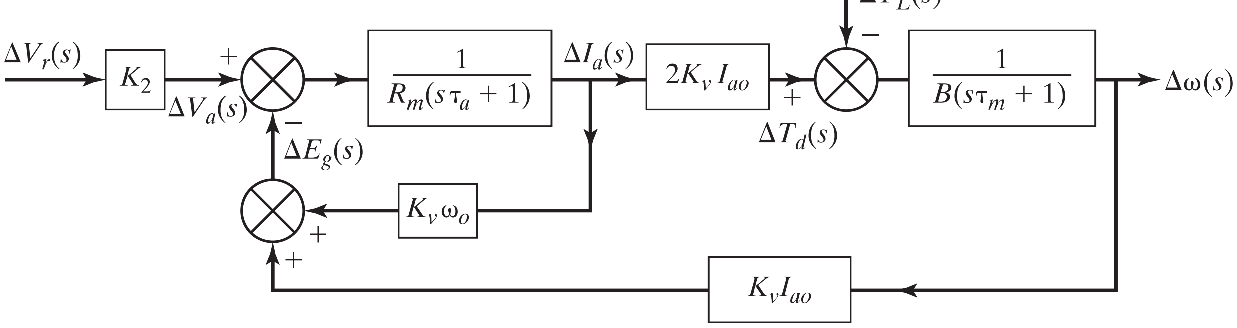
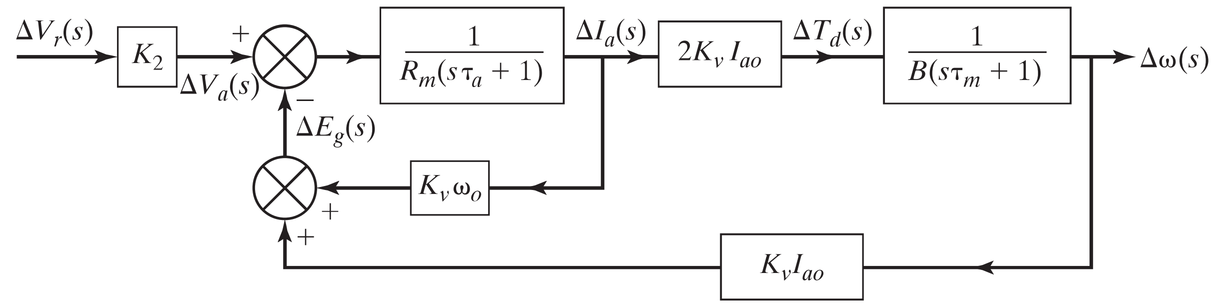
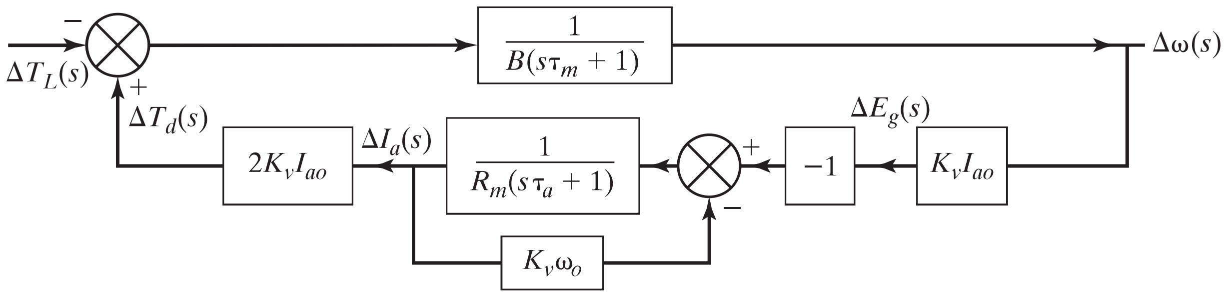


FIGURE 14.27

Open-loop block diagram of dc–dc converter-fed dc series drive.



(a) Step change in voltage



(b) Step change in torque

FIGURE 14.28

Block diagram for reference voltage and load torque distributions.

reference voltage or load torque can result in a change of speed. The block diagram for a change in reference voltage is shown in Figure 14.28a and that for a change in load torque is shown in Figure 14.28b.

14.7.4 Converter Control Models

We can notice from Eq. (14.32) that the average output voltage of a dc–dc converter is directly proportional to the duty cycle k which is a direct function of the control voltage. The gain of a dc–dc converter can be expressed as

$$K_r = k = \frac{V_c}{V_{cm}} \quad (\text{for dc–dc converter}) \tag{14.83}$$

where V_c is the control signal voltage (say 0 to 10 V) and V_{cm} is the maximum value of the control signal voltage (10 V).

The average output voltage of a single-phase converter as in Eq. (14.20) is a cosine function of the delay angle α. The control input signal can be modified to determine the delay angle

$$\alpha = \cos^{-1}\left(\frac{V_c}{V_{cm}}\right) = \cos^{-1}(V_{cn}) \tag{14.84}$$

Using Eq. (14.84), the average output voltage of a single-phase converter in Eq. (14.20) can be expressed as

$$V_a = \frac{2V_m}{\pi}\cos\alpha = \frac{2V_m}{\pi}\cos[\cos^{-1}(V_{cn})]$$

$$= \left[\frac{2V_m}{\pi}\right]V_{cn} = \left[\frac{2V_m}{\pi V_{cm}}\right]V_c = K_c V_c \tag{14.85}$$

where K_c is the gain of the single-phase converter as

$$K_r = \frac{2V_m}{\pi V_{cm}} = \frac{2 \times \sqrt{2}}{\pi V_{cm}}V_s = 0.9\frac{V_s}{V_{cm}} \quad \text{(for single-phase converter)} \tag{14.86}$$

where $V_s = V_m/\sqrt{2}$ is the rms value of the single-phase ac supply voltage.

Using Eq. (14.84), the average output voltage of a three-phase converter in Eq. (14.27) can be expressed as

$$V_a = \frac{3\sqrt{3}\,V_m}{\pi}\cos\alpha = \frac{3\sqrt{3}\,V_m}{\pi}\cos[\cos^{-1}(V_{cn})]$$

$$= \left[\frac{3\sqrt{3}\,V_m}{\pi}\right]V_{cn} = \left[\frac{3\sqrt{3}\,V_m}{\pi V_{cm}}\right]V_c = K_r V_c \tag{14.87}$$

where K_r is the gain of the three-phase converter as

$$K_r = \frac{3\sqrt{3}\,V_m}{\pi V_{cm}} = \frac{3\sqrt{3}\sqrt{2}}{\pi V_{cm}}V_s = 2.339\frac{V_s}{V_{cm}} \quad \text{(for three-phase converter)} \tag{14.88}$$

where $V_s = V_m/\sqrt{2}$ is the rms value of the ac supply voltage per phase.

Therefore, a converter can be modeled with a transfer function $G_c(s)$ of a certain gain and phase delay as described by

$$G_c(s) = K_r e^{-\tau_r} \tag{14.89}$$

which can be approximated as a first-order lag function given by

$$G_r(s) = \frac{K_r}{1 + s\tau_r} \tag{14.90}$$

where τ_r is the time delay of the sampling interval. Once a switching device is turned on, its gating signal cannot be changed. There is a delay until the next device is gated and a corrective action is implemented. The delay time is generally half of the interval between two switching devices.

Therefore, the delay time for a frequency of f_s can be determined from

$$\tau_r = \frac{360/(2 \times 6)}{360} T_s = \frac{1}{12} \times \frac{1}{f_s} \quad \text{(for three-phase converter)} \tag{14.91a}$$

$$= \frac{360/(2 \times 4)}{360} T_s = \frac{1}{8} \times \frac{1}{f_s} \quad \text{(for single-phase converter)} \tag{14.91b}$$

$$= \frac{360/(2 \times 1)}{360} T_s = \frac{1}{2} \times \frac{1}{f_s} \quad \text{(for dc−dc converter)} \tag{14.91c}$$

For $f_s = 60\,\text{Hz}, \tau_r = 1.389\,\text{ms}$ for a three-phase converter, $\tau_r = 2.083\,\text{ms}$ for a single-phase converter, and $\tau_r = 8.333\,\text{ms}$ for a dc–dc converter.

14.7.5 Closed-Loop Transfer Function

Once the models for motors are known, feedback paths can be added to obtain the desired output response. To change the open-loop arrangement in Figure 14.23 into a closed-loop system, a speed sensor is connected to the output shaft. The output of the sensor, which is proportional to the speed, is amplified by a factor of K_1 and is compared with the reference voltage V_r to form the error voltage V_e. The complete block diagram is shown in Figure 14.29.

The closed-loop step response due to a change in reference voltage can be found from Figure 14.24 with $T_L = 0$. The transfer function becomes

$$\frac{\omega(s)}{V_r(s)} = \frac{K_2 K_v I_f/(R_m B)}{s^2(\tau_a \tau_m) + s(\tau_a + \tau_m) + 1 + [(K_v I_f)^2 + K_1 K_2 K_v I_f]/R_m B} \tag{14.92}$$

The response due to a change in the load torque T_L can also be obtained from Figure 14.29 by setting V_r to zero. The transfer function becomes

$$\frac{\omega(s)}{T_L(s)} = -\frac{(1/B)(s\tau_a + 1)}{s^2(\tau_a \tau_m) + s(\tau_a + \tau_m) + 1 + [(K_v I_f)^2 + K_1 K_2 K_v I_f]/R_m B} \tag{14.93}$$

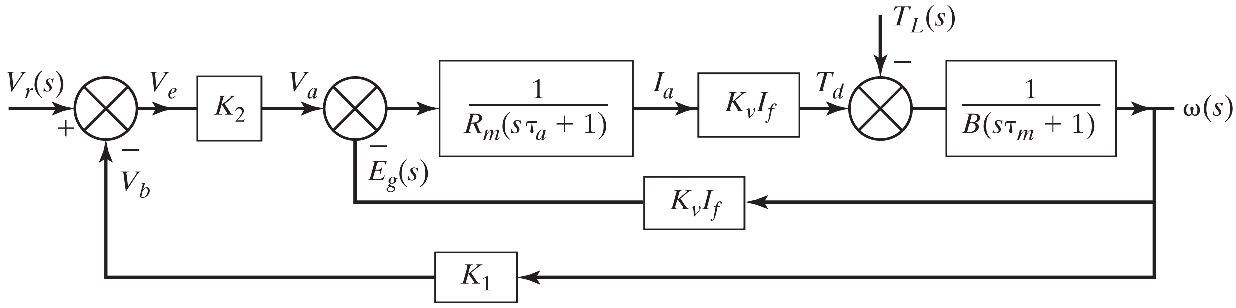


FIGURE 14.29

Block diagram for closed-loop control of separately excited dc motor.

Using the final value theorem, the steady-state change in speed $\Delta\omega$, due to a step change in control voltage ΔV_r and a step change in load torque ΔT_L, can be found from Eqs. (14.92) and (14.93), respectively, by substituting $s = 0$.

$$\Delta\omega = \frac{K_2 K_v I_f}{R_m B + (K_v I_f)^2 + K_1 K_2 K_v I_f} \Delta V_r \tag{14.94}$$

$$\Delta\omega = -\frac{R_m}{R_m B + (K_v I_f)^2 + K_1 K_2 K_v I_f} \Delta T_L \tag{14.95}$$

Example 14.13 Finding the Speed and Torque Response of a Converter-Fed Drive

A 50-kW, 240-V, 1700-rpm separately excited dc motor is controlled by a converter, as shown in the block diagram Figure 14.29. The field current is maintained constant at $I_f = 1.4$ A and the machine back emf constant is $K_v = 0.91$ V/A rad/s. The armature resistance is $R_m = 0.1\ \Omega$ and the viscous friction constant is $B = 0.3$ N·m/rad/s. The amplification of the speed sensor is $K_1 = 95$ mV/rad/s and the gain of the power controller is $K_2 = 100$. (a) Determine the rated torque of the motor. (b) Determine the reference voltage V_r to drive the motor at the rated speed. (c) If the reference voltage is kept unchanged, determine the speed at which the motor develops the rated torque. (d) If the load torque is increased by 10% of the rated value, determine the motor speed. (e) If the reference voltage is reduced by 10%, determine the motor speed. (f) If the load torque is increased by 10% of the rated value and the reference voltage is reduced by 10%, determine the motor speed. (g) If there was no feedback in an open-loop control, determine the speed regulation for a reference voltage of $V_r = 2.31$ V. (h) Determine the speed regulation with a closed-loop control.

Solution

$I_f = 1.4$ A, $K_v = 0.91$ V/A rad/s, $K_1 = 95$ mV/rad/s, $K_2 = 100$, $R_m = 0.1\ \Omega$, $B = 0.3$ N·m/rad/s, and $\omega_{rated} = 1700\ \pi/30 = 178.02$ rad/s.

a. The rated torque is $T_L = 50{,}000/178.02 = 280.87$ N·m.

b. Because $V_a = K_2 V_r$, for open-loop control Eq. (14.70) gives

$$\frac{\omega}{V_a} = \frac{\omega}{K_2 V_r} = \frac{K_v I_f}{R_m B + (K_v I_f)^2} = \frac{0.91 \times 1.4}{0.1 \times 0.3 + (0.91 \times 1.4)^2} = 0.7707$$

At rated speed,

$$V_a = \frac{\omega}{0.7707} = \frac{178.02}{0.7707} = 230.98 \text{ V}$$

and feedback voltage,

$$V_b = K_1 \omega = 95 \times 10^{-3} \times 178.02 = 16.912 \text{ V}$$

With closed-loop control, $(V_r - V_b)K_2 = V_a$ or $(V_r - 16.912) \times 100 = 230.98$, which gives the reference voltage, $V_r = 19.222$ V.

c. For $V_r = 19.222$ V and $\Delta T_L = 280.87$ N · m, Eq. (14.95) gives

$$\Delta\omega = -\frac{0.1 \times 280.86}{0.1 \times 0.3 + (0.91 \times 1.4)^2 + 95 \times 10^{-3} \times 100 \times 0.91 \times 1.4}$$

$$= -2.04 \text{ rad/s}$$

The speed at rated torque,

$$\omega = 178.02 - 2.04 = 175.98 \text{ rad/s} \quad \text{or} \quad 1680.5 \text{ rpm}$$

d. $\Delta T_L = 1.1 \times 280.87 = 308.96$ N · m and Eq. (14.95) gives

$$\Delta\omega = -\frac{0.1 \times 308.96}{0.1 \times 0.3 + (0.91 \times 1.4)^2 + 95 \times 10^{-3} \times 100 \times 0.91 \times 1.4}$$

$$= -2.246 \text{ rad/s}$$

The motor speed

$$\omega = 178.02 - 2.246 = 175.774 \text{ rad/s} \quad \text{or} \quad 1678.5 \text{ rpm}$$

e. $\Delta V_r = -0.1 \times 19.222 = -1.9222$ V and Eq. (14.94) gives the change in speed,

$$\Delta\omega = -\frac{100 \times 0.91 \times 1.4 \times 1.9222}{0.1 \times 0.3 + (0.91 \times 1.4)^2 + 95 \times 10^{-3} \times 100 \times 0.91 \times 1.4}$$

$$= -17.8 \text{ rad/s}$$

The motor speed is

$$\omega = 178.02 - 17.8 = 160.22 \text{ rad/s} \quad \text{or} \quad 1530 \text{ rpm}$$

f. The motor speed can be obtained by using superposition:

$$\omega = 178.02 - 2.246 - 17.8 = 158 \text{ rad/s} \quad \text{or} \quad 1508.5 \text{ rpm}$$

g. $\Delta V_r = 2.31$ V and Eq. (14.70) gives

$$\Delta\omega = \frac{100 \times 0.91 \times 1.4 \times 2.31}{0.1 \times 0.3 + (0.91 \times 1.4)^2} = 178.02 \text{ rad/s} \quad \text{or} \quad 1700 \text{ rpm}$$

and the no-load speed is $\omega = 178.02$ rad/s or 1700 rpm. For full load, $\Delta T_L = 280.87$ N · m, Eq. (14.71) gives

$$\Delta\omega = -\frac{0.1 \times 280.87}{0.1 \times 0.3 + (0.91 \times 1.4)^2} = -16.99 \text{ rad/s}$$

and the full-load speed

$$\omega = 178.02 - 16.99 = 161.03 \text{ rad/s} \quad \text{or} \quad 1537.7 \text{ rpm}$$

The speed regulation with open-loop control is

$$\frac{1700 - 1537.7}{1537.7} = 10.55\%$$

h. Using the speed from (c), the speed regulation with closed-loop control is

$$\frac{1700 - 1680.5}{1680.5} = 1.16\%$$

Note: By closed-loop control, the speed regulation is reduced by a factor of approximately 10, from 10.55% to 1.16%.

14.7.6 Closed-Loop Current Control

The block diagram of a dc machine contains an inner loop due to the induced emf *e,* as shown in Figure 14.30a. B_1 and B_l are the viscous frictions for the motor and the load, respectively. H_c is the gain of the current feedback; a low-pass filter with a time constant of less than 1 ms may be needed in some applications. The inner current loop will cross this back emf loop. The interactions of these loops can be decoupled by moving the K_b block to the I_a feedback signal, as shown in Figure 14.30b. This will allow splitting the transfer function between the speed ω and the input voltage V_a into two cascaded transfer functions (a) between the speed and the armature current, and (b) then between the armature current and the input voltage. That is,

$$\frac{\omega(s)}{V_a(s)} = \frac{\omega(s)}{I_a(s)} \times \frac{I_a(s)}{V_a(s)} \tag{14.96}$$

The blocks in Figure 14.30b can be simplified to obtain the following transfer function relationships

$$\frac{\omega(s)}{I_a(s)} = \frac{K_b}{B(1 + s\tau_m)} \tag{14.97}$$

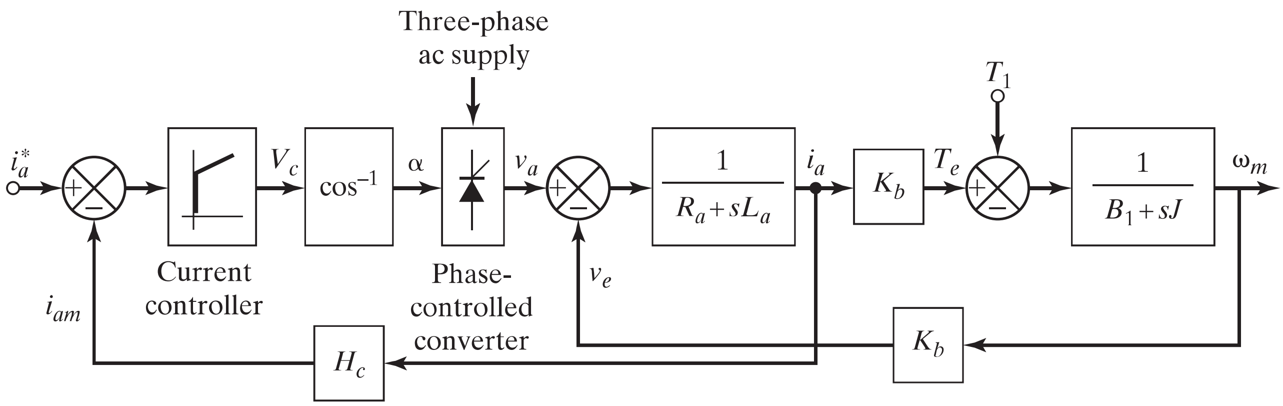
$$\frac{I_a(s)}{V_a(s)} = K_m\frac{1 + s\tau_m}{(1 + s\tau_1)(1 + s\tau_2)} \tag{14.98}$$

where

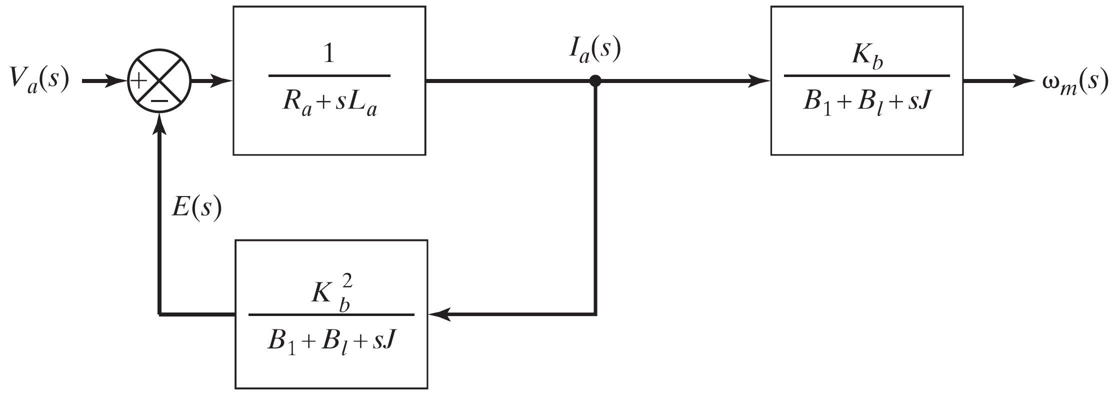
$$K_m = \frac{B}{K_b^2 + R_mB} \tag{14.99}$$

$$-\frac{1}{\tau_1}; -\frac{1}{\tau_2} = \frac{-\left(\frac{R_m}{L_m} + \frac{B}{J}\right) \pm \sqrt{\left(\frac{R_m}{L_m} + \frac{B}{J}\right)^2 - 4\left(\frac{R_mB + K_b^2}{JL_m}\right)}}{2} \tag{14.100}$$

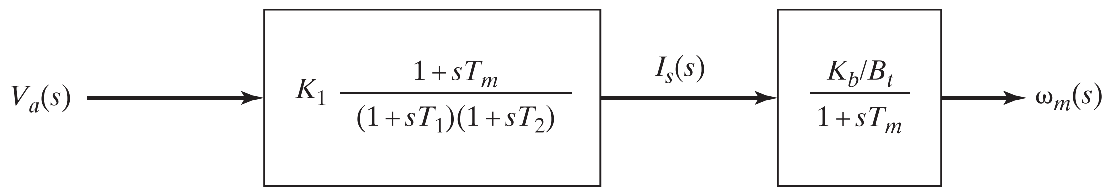
Two block representation of the open-loop motor transfer function between the output speed and the input voltage is shown in Figure 14.30c, where total viscous friction $B_t = B_1 + B_l$.



(a) Inner current loop



(b) Manipulation of moving the block K_b for I_a feedback



(c) Open-loop motor transfer function

FIGURE 14.30

Converter-fed dc motor with current-control loop.

The overall closed-loop system with a current feedback loop is shown in Figure 14.31a, where B_t is the total viscous friction of the motor and the load. The converter can be represented by Eq. (14.90). Using proportional-integral type of control, the transfer functions of the current controller $G_c(s)$ and the speed controller $G_s(s)$ can be represented as

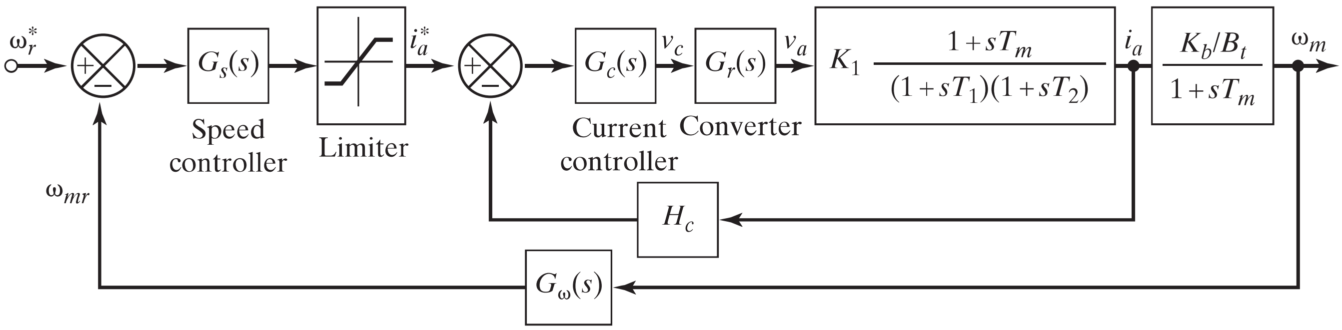
$$G_c(s) = \frac{K_c(1 + s\tau_c)}{s\tau_c} \tag{14.101}$$

$$G_s(s) = \frac{K_s(1 + s\tau_s)}{s\tau_c} \tag{14.102}$$

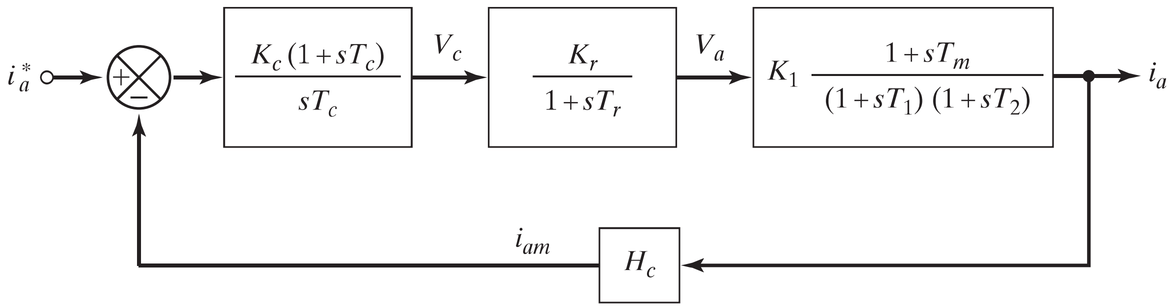
where K_c and K_s are the gains of the current and speed controllers, respectively, and τ_c and τ_s are the time constants of the current and speed controllers, respectively. A dc tachogenerator along with a low-pass fitter of a time constant less than 10 ms is often required. The transfer function of the speed feedback filter can be expressed as

$$G_{\omega}(s) = \frac{K_{\omega}}{1 + s\tau_{\omega}} \tag{14.103}$$

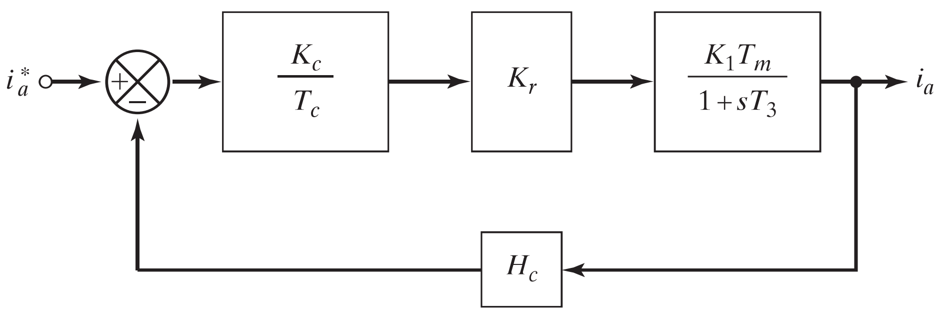
where K_{ω} and τ_{ω} are the gain and time constants of the speed feedback loop. Figure 14.31b shows the block diagrams with the current-control loop. i_{am} is the feedback armature-motor current. The simplified diagram is shown in Figure 14.31c.



(a) Motor drive with current-control loop



(b) Current-control loop



(c) Simplified current-control loop

FIGURE 14.31

Motor drive with current- and speed-control loops.

Figure 14.29 uses a speed feedback only. In practice, the motor is required to operate at a desired speed, but it has to meet the load torque, which depends on the armature current. While the motor is operating at a particular speed, if a load is applied suddenly, the speed falls and the motor takes time to come up to the desired speed. A speed feedback with an inner current loop, as shown in Figure 14.32, provides faster response to any disturbances in speed command, load torque, and supply voltage.

The current loop is used to cope with a sudden torque demand under transient condition. The output of the speed controller e_c is applied to the current limiter, which sets the current reference $I_{a(\text{ref})}$ for the current loop. The armature current I_a is sensed by a current sensor, filtered normally by an active filter to remove ripple, and compared with the current reference $I_{a(\text{ref})}$. The error current is processed through a current controller whose output v_c adjusts the firing angle of the converter and brings the motor speed to the desired value.

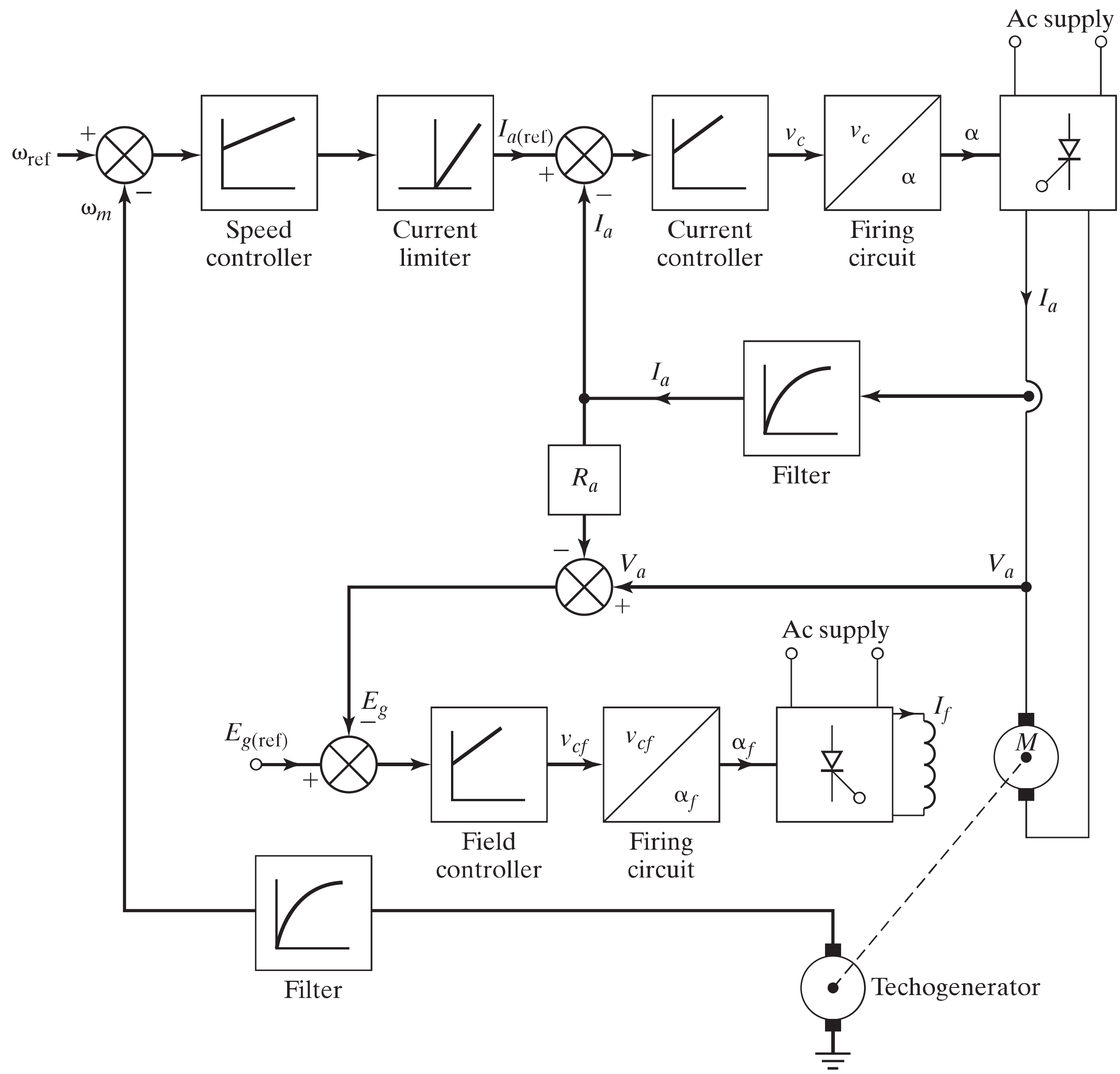


FIGURE 14.32

Closed-loop speed control with inner current loop and field weakening.

Any positive speed error caused by an increase in either speed command or load torque demand can produce a high reference current $I_{a(\text{ref})}$. The motor accelerates to correct the speed error, and finally settles at a new $I_{a(\text{ref})}$, which makes the motor torque equal to the load torque, resulting in a speed error close to zero. For any large positive speed error, the current limiter saturates and limits the reference current $I_{a(\text{ref})}$ to a maximum value $I_{a(\max)}$. The speed error is then corrected at the maximum permissible armature current $I_{a(\max)}$ until the speed error becomes small and the current limiter comes out of saturation. Normally, the speed error is corrected with I_a less than the permissible value $I_{a(\max)}$.

The speed control from zero to base speed is normally done at the maximum field by armature voltage control, and control above the base speed should be done by field weakening at the rated armature voltage. In the field control loop, the back emf $E_g(= V_a - R_a I_a)$ is compared with a reference voltage $E_{g(\text{ref})}$, which is generally between 0.85 to 0.95 of the rated armature voltage. For speeds below the base speed, the field error e_f is large and the field controller saturates, thereby applying the maximum field voltage and current.

When the speed is close to the base speed, V_a is almost near to the rated value and the field controller comes out of saturation. For a speed command above the base speed, the speed error causes a higher value of V_a. The motor accelerates, the back emf E_g increases, and the field error e_f decreases. The field current then decreases and the motor speed continues to increase until the motor speed reaches the desired speed. Thus, the speed control above the base speed is obtained by the field weakening while the armature terminal voltage is maintained at near the rated value. In the field weakening mode, the drive responds very slowly due to the large field time constant. A full converter is normally used in the field, because it has the ability to reverse the voltage, thereby reducing the field current much faster than a semiconverter.

14.7.7 Design of Current Controller

The loop gain function of the motor drive in Figure 14.31b is given by

$$G(s)H(s) = \left(\frac{K_m K_c K_r H_c}{\tau_c}\right)\frac{(1 + s\tau_c)(1 + s\tau_m)}{s(1 + s\tau_1)(1 + s\tau_2)(1 + s\tau_r)} \tag{14.104}$$

In order to reduce the system to a second order, the following assumptions can be made for practical motor drives:

$$1 + s\tau_m \approx s\tau_m; \tau_1 > \tau_2 > \tau_r \text{ and } \tau_2 = \tau_c$$

With these assumptions, Eq. (14.104) can be simplified [13] to

$$G(s)H(s) = \frac{K}{(1 + s\tau_1)(1 + s\tau_r)} \tag{14.105}$$

where

$$K = \frac{K_m K_c K_r H_c \tau_m}{\tau_c} \tag{14.106}$$

The characteristic equation of the loop gain in Eq. (14.105) is given by

$$1 + G(s)H(s) = (1 + s\tau_1)(1 + s\tau_r) + K = 0 \tag{14.107}$$

This gives the natural frequency ω_n and damping factor ξ of the current-control loop as

$$\omega_n = \sqrt{\frac{1 + K}{\tau_1 \tau_r}} \tag{14.108}$$

$$\zeta = \frac{\dfrac{\tau_1 + \tau_r}{\tau_1 \tau_r}}{2\omega_n} \tag{14.109}$$

Setting the damping ratio $\xi = 0.707$ for critically damped, assuming $K \gg 1$ and also $\tau_1 > \tau_r$, the gain of the current controller can be expressed as [13]

$$K_c = \frac{1}{2}\frac{\tau_1 \tau_c}{\tau_r}\frac{1}{K_m K_r H_c \tau_m} \tag{14.110}$$

14.7.8 Design of Speed Controller

The design of the speed controller can be simplified by replacing the second-order model of the current loop with an approximate first-order model. The current loop is approximated by adding the time delay τ_r of the converter to the time delay τ_1 of the motor, as shown in Figure 14.31c. The transfer function of the current-control loop in Figure 14.31c can be expressed as [13]

$$\frac{I_a(s)}{I_a^*(s)} = \frac{K_i}{(1 + s\tau_i)} \tag{14.111}$$

where

$$\tau_i = \frac{\tau_3}{1 + K_1} \tag{14.112}$$

$$\tau_3 = \tau_1 + \tau_r \tag{14.113}$$

$$K_i = \frac{1}{H_c}\left(\frac{K_1}{1 + K_1}\right) \tag{14.114}$$

$$K_1 = \frac{K_m K_c K_r H_c \tau_m}{\tau_c} \tag{14.115}$$

Replacing the speed controller with its transfer function in Eq. (14.102) and approximating current-control loop in Eq. (14.111), the block diagram of the outer speed-control loop is shown in Figure 14.33. The loop gain function is expressed as

$$G(s)H(s) = \left(\frac{K_s K_i K_b K_\omega}{B\tau_s}\right)\frac{(1 + s\tau_s)}{s(1 + s\tau_i)(1 + s\tau_m)(1 + s\tau_\omega)} \tag{14.116}$$

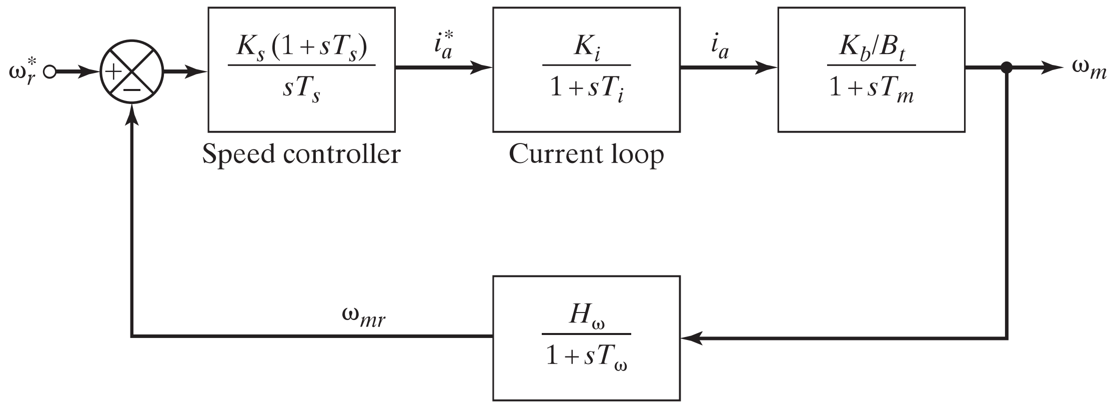


FIGURE 14.33

Outer speed-control loop.

Assuming $1 + s\tau_m \approx s\tau_m$ and combining the delay time τ_s of the speed controller with the delay time τ_ω of the speed feedback filter to an equivalent delay time $t_e = \tau_s + \tau_\omega$, Eq. (14.116) can be approximated to

$$G(s)H(s) = K_2\left(\frac{K_s}{\tau_s}\right)\frac{(1 + s\tau_s)}{s^2(1 + s\tau_e)} \tag{14.117}$$

where

$$\tau_e = \tau_i + \tau_\omega \tag{14.118}$$

$$K_\omega = \frac{K_i K_b H_\omega}{B\tau_m} \tag{14.119}$$

The closed-loop transfer function of the speed in response to speed reference signal is given by

$$\frac{\omega(s)}{\omega_r^*(s)} = \frac{1}{K_\omega}\left[\frac{\frac{K_\omega K_s}{T_s}(1 + s\tau_s)}{s^3\tau_e + s^2 + sK_\omega K_s + \frac{K_\omega K_s}{T_s}}\right] \tag{14.120}$$

Equation (14.120) can be optimized by making the magnitude of its denominator such that the coefficients of ω2 and ω4 in the frequency domain are zero. This should widen the bandwidth operating over a wide frequency range. It can be shown that the gain K_s and the time constant τ_s of the speed controller under the optimum conditions are given by [13]

$$K_s = \frac{1}{2K_\omega\tau_e} \tag{14.121}$$

$$\tau_s = 4\tau_e \tag{14.122}$$

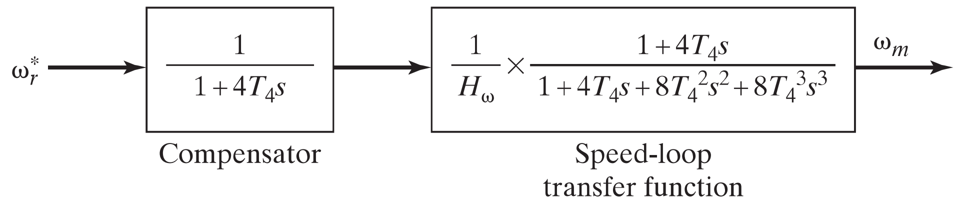


FIGURE 14.34

Adding a pole-zero compensating network.

Substituting K_s and τ_s into Eq. (14.120) gives the optimized closed-loop transfer function of the speed in response to speed input signal as

$$\frac{\omega(s)}{\omega_r^*(s)} = \frac{1}{K_\omega}\left[\frac{(1 + 4s\tau_e)}{1 + 4s\tau_e + 8s^2\tau_e^2 + 8s^3\tau_e^3}\right] \tag{14.123}$$

It can be shown that the corner frequencies for the open-loop gain function $H(s)G(s)$ are $1/(4\tau_e)$ and $1/\tau_e$. The gain crossover occurs at $1/(2\tau_e)$ at a slope of the magnitude response—20 dB/decade. As a result, the transient response should exhibit the most desirable characteristic for good dynamic behavior. The transient response is given by [13]

$$\omega_r(t) = \frac{1}{K_\omega}\left[1 + e^{\frac{-t}{2\tau_e}} - 2e^{\frac{-t}{4\tau_e}}\cos\left(\frac{\sqrt{3}t}{4\tau_e}\right)\right] \tag{14.124}$$

This gives a rise time of $3.1\tau_e$, a maximum overshoot of 43.4%, and a settling time of $16.5\tau_e$. The high overshoot can be reduced by adding a compensating network with a pole-zero in the speed feedback path, as shown in Figure 14.34. The compensated closed-loop transfer function of the speed in response to speed input signal is

$$\frac{\omega(s)}{\omega_r^*(s)} = \frac{1}{K_\omega}\left[\frac{1}{1 + 4s\tau_e + 8s^2\tau_e^2 + 8s^3\tau_e^3}\right] \tag{14.125}$$

The corresponding compensated transient response is given by

$$\omega_r(t) = \frac{1}{K_\omega}\left[1 + e^{\frac{-t}{4\tau_e}} - \frac{2}{\sqrt{3}}e^{\frac{-t}{4\tau e}}\sin\left(\frac{\sqrt{3}t}{4\tau_e}\right)\right] \tag{14.126}$$

This gives a rise time of 7.6 τ_e, a maximum overshoot of 8.1%, and a settling time of 13.3 τ_e. It should be noted that the overshoot has been reduced to approximately 20% of the value in Eq. (14.20), and the settling time has come down by 19%. But the rise time has increased and the designer has to make a compromise on the rise time and the overshoot.

Example 14.14 Determining the Optimized Gains and Time Constants of the Current and Speed Loop Controllers

The motor parameters of a converter-fed dc motor are 220 V, 6.4 A, 1570 rpm, $R_m = 6.5\ \Omega$, $J = 0.06\,\text{kg-m}^2$, $L_m = 67\,\text{mH}$, $B = 0.087\,\text{N-rn/rad/s}$, $K_b = 1.24\,\text{V/rad/s}$. The converter is supplied from a Y-connected 230 V, three-phase ac source at 60 Hz. The converter can be assumed linear and its maximum control input voltage is $V_{cm} = \pm 10\,\text{V}$. The tachogenerator has the transfer function $G\omega(s) = 0.074/(1 + 0.002s)$. The speed reference voltage has a maximum of 10 V. The maximum permissible motor current is 20 A. Determine (a) the gain K_r and time constant τ_r of the converter, (b) the current feedback gain H_c, (c) the motor time constant τ_1, τ_2, and τ_m, (d) the gain K_c and the time constant τ_c of the current controller, (e) the gain K_i and the time constant τ_i of the simplified current loop, and (f) the optimized gain K_s and the time constant τ_s of the speed controller.

Solution

$V_{dc} = 220\,\text{V}$, $I_{dc} = 6.4\,\text{A}$, $N = 1570$ rpm, $R_m = 6.5\,\Omega$, $J = 0.06\,\text{kg-m}^2$, $L_m = 67\,\text{mH}$, $B = 0.087$ N-m/rad/s, $K_b = 1.24\,\text{V/rad/s}$, $V_L = 220\,\text{V}$, $f_s = 60\,\text{Hz}$, $V_{cm} = \pm 10\,\text{V}$ 20 A, $I_{a(\max)} = 20\,\text{A}$, $K_\omega = 0.074$, $t_\omega = 0.002\,\text{s}$.

a. The phase voltage is $V_s = \dfrac{V_L}{\sqrt{3}} = \dfrac{220}{\sqrt{3}} = 127.02\,\text{V}$

The maximum dc voltage is $V_{dc(\max)} = K_r V_{cm} = 29.71 \times 10 = 297.09\,\text{V}$

The converter control voltage is $V_c = \dfrac{V_{dc}}{V_{dc(\max)}} V_{cm} = \dfrac{220 \times 110}{297.09} = 7.41\,\text{V}$

Using Eq. (14.88), $K_s = \dfrac{2.339 V_s}{V_{cm}} = \dfrac{2.339 \times 127.02}{10} = 29.71\,\text{V/V}$

Using Eq. (14.91a) $\tau_r = \dfrac{1}{12 f_s} = \dfrac{1}{12 \times 60} = 1.39\,\text{ms}$

b. The current feedback gain is $H_c = \dfrac{V_c}{I_{a(max)}} = \dfrac{7.41}{20} = 0.37\,\text{V/A}$

c. Using $B_t = B$ and Eq. (14.99), $K_m = \dfrac{B}{K_b^2 + R_m B} = \dfrac{0.087}{1.24^2 + 6.5 \times 0.087} = 0.04$

Using Eq. (14.100),

$$r_1 = -5.64; \quad \tau_1 = \frac{-1}{r_1} = 0.18\,\text{s}$$

$$r_2 = -92.83; \quad \tau_2 = \frac{-1}{r_2} = 0.01\,\text{s}$$

$$\tau_m = \frac{J}{B} = \frac{0.06}{0.087} = 0.69\,\text{s}$$

d. The time constant of the current controller is $\tau_c = \tau_2 = 0.01\,\text{s}$

Using Eq. (14.110), $K_c = \dfrac{\tau_1 \tau_c}{2\tau_r}\left(\dfrac{1}{K_m K_r H_c \tau_m}\right) = \dfrac{0.18 \times 0.01}{0.04 \times 29.71 \times 0.37 \times 0.69} = 2.19$

Using Eqs (14.112) to (14.115),

$$K_1 = \frac{K_m K_c K_r H_c \tau_m}{\tau_c} = \frac{0.4 \times 2.19 \times 29.71 \times 0.37 \times 0.69}{0.01} = 63.88$$

$$K_i = \frac{1}{H_c}\left(\frac{K_1}{1 + K_1}\right) = \frac{1}{0.37}\left(\frac{63.88}{1 + 63.88}\right) = 2.66$$

$$\tau_3 = \tau_1 + \tau_r = 0.18 + 0.00139 = 0.18\,\text{s}$$

$$\tau_i = \frac{\tau_3}{1 + K_1} = \frac{0.18}{1 + 63.88} = 2.76\,\text{ms}$$

e. Using Eqs (14.118) and (14.119),

$$\tau_e = \tau_i + \tau_\omega = 2.76 \times 10^{-3} + 2 \times 10^{-3} = 4.76\,\text{ms}$$

$$K_\omega = \frac{K_i K_b H_\omega}{B\tau_m} = \frac{2.66 \times 1.24 \times 0.074}{0.087 \times 0.69} = 4.07$$

Using Eqs (14.121) and (14.122)

$$K_s = \frac{1}{2K_\omega \tau_e} = \frac{1}{2 \times 4.07 \times 4.76 \times 10^{-3}} = 25.85$$

$$\tau_s = 4\tau_e = 4 \times 4.76 \times 10^{-3} = 19.02\,\text{ms}$$

14.7.9 Dc–dc Converter-Fed Drive

Dc–dc converter-fed dc drives can operate from a rectified dc supply or a battery. These can also operate in one, two, or four quadrants, offering a few choices to meet application requirements [9]. Servo drive systems normally use the full four-quadrant converter, as shown in Figure 14.35, which allows bidirectional speed control with regenerative breaking capabilities. For forward driving, the transistors T_1 and T_4 and diode D_2 are used as a buck converter that supplies a variable voltage, v_a, to the armature given by

$$v_a = \delta V_{DC} \tag{14.127}$$

where V_{DC} is the dc supply voltage to the converter and δ is the duty cycle of the transistor T_1.

During regenerative braking in the forward direction, transistor T_2 and diode D_4 are used as a boost converter, which regulates the breaking current through the motor by automatically adjusting the duty cycle of T_2. The energy of the overhauling motor now returns to the dc supply through diode D_1, aided by the motor back emf and the dc supply. The braking converter, composed of T_2 and D_1, may be used to maintain the regenerative braking current at the maximum allowable level right down to zero speed. Figure 14.36 shows a typical acceleration–deceleration profile of a dc–dc converter-fed drive under the closed-loop control.

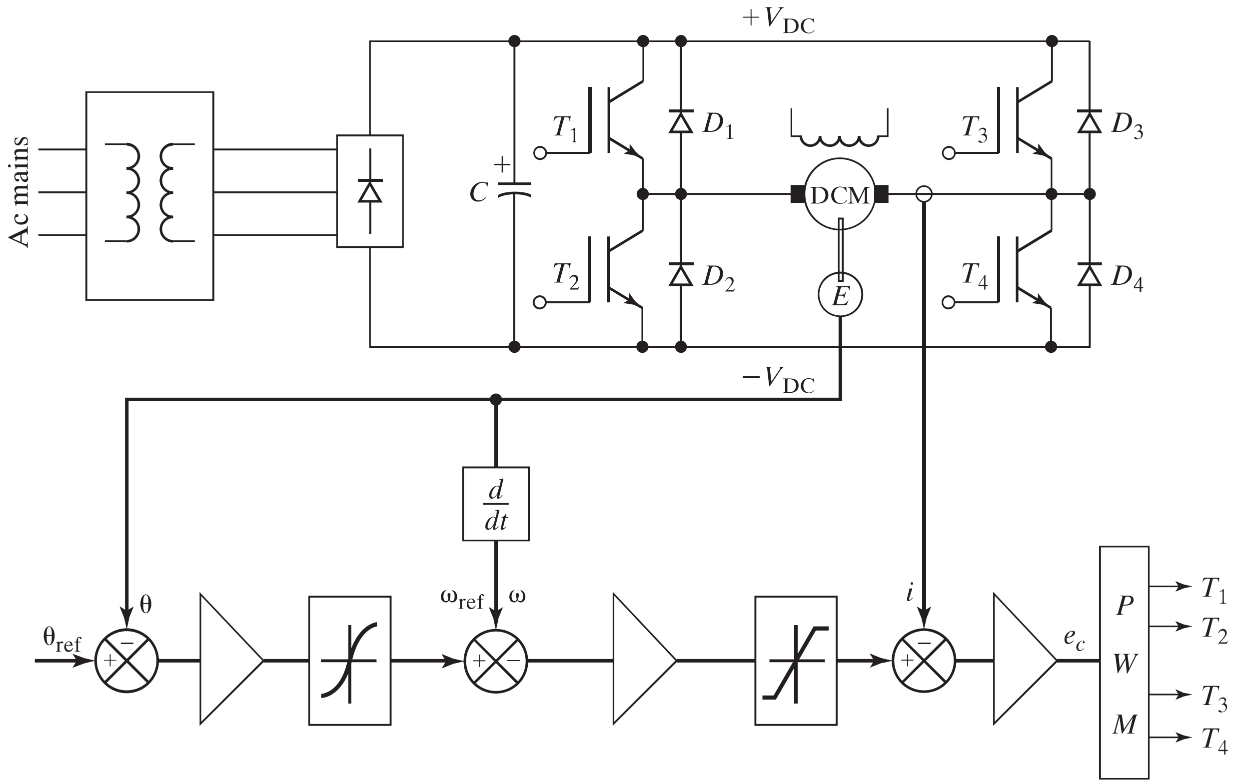


FIGURE 14.35

Dc–dc converter-fed speed and position control system with an IGBT bridge.

14.7.10 Phase-Locked-Loop Control

For precise speed control of servo systems, closed-loop control is normally used. The speed, which is sensed by analog sensing devices (e.g., tachometer), is compared with the reference speed to generate the error signal and to vary the armature voltage of the motor. These analog devices for speed sensing and comparing signals are not ideal and the speed regulation is more than 0.2%. The speed regulator can be improved if digital phase-locked-loop (PLL) control is used [10]. The block diagram of a converter-fed dc motor drive with PLL control is shown in Figure 14.37a and the transfer function block diagram is shown in Figure 14.37b.

In a PLL control system, the motor speed is converted to a digital pulse train by using a speed encoder. The output of the encoder acts as the speed feedback signal of frequency f_0. The phase detector compares the reference pulse train (or frequency) f_r with the feedback frequency f_0 and provides a pulse-width-modulated (PWM) output voltage V_e that is proportional to the difference in phases and frequencies of the reference and feedback pulse trains. The phase detector (or comparator) is available in integrated circuits. A low-pass loop filter converts the pulse train V_e to a continuous dc level V_c, which varies the output of the power converter and in turn the motor speed.

When the motor runs at the same speed as the reference pulse train, the two frequencies would be synchronized (or locked) together with a phase difference. The output of the phase detector would be a constant voltage proportional to the phase difference and the steady-state motor speed would be maintained at a fixed value irrespective of the load on the motor. Any disturbances contributing to the speed change

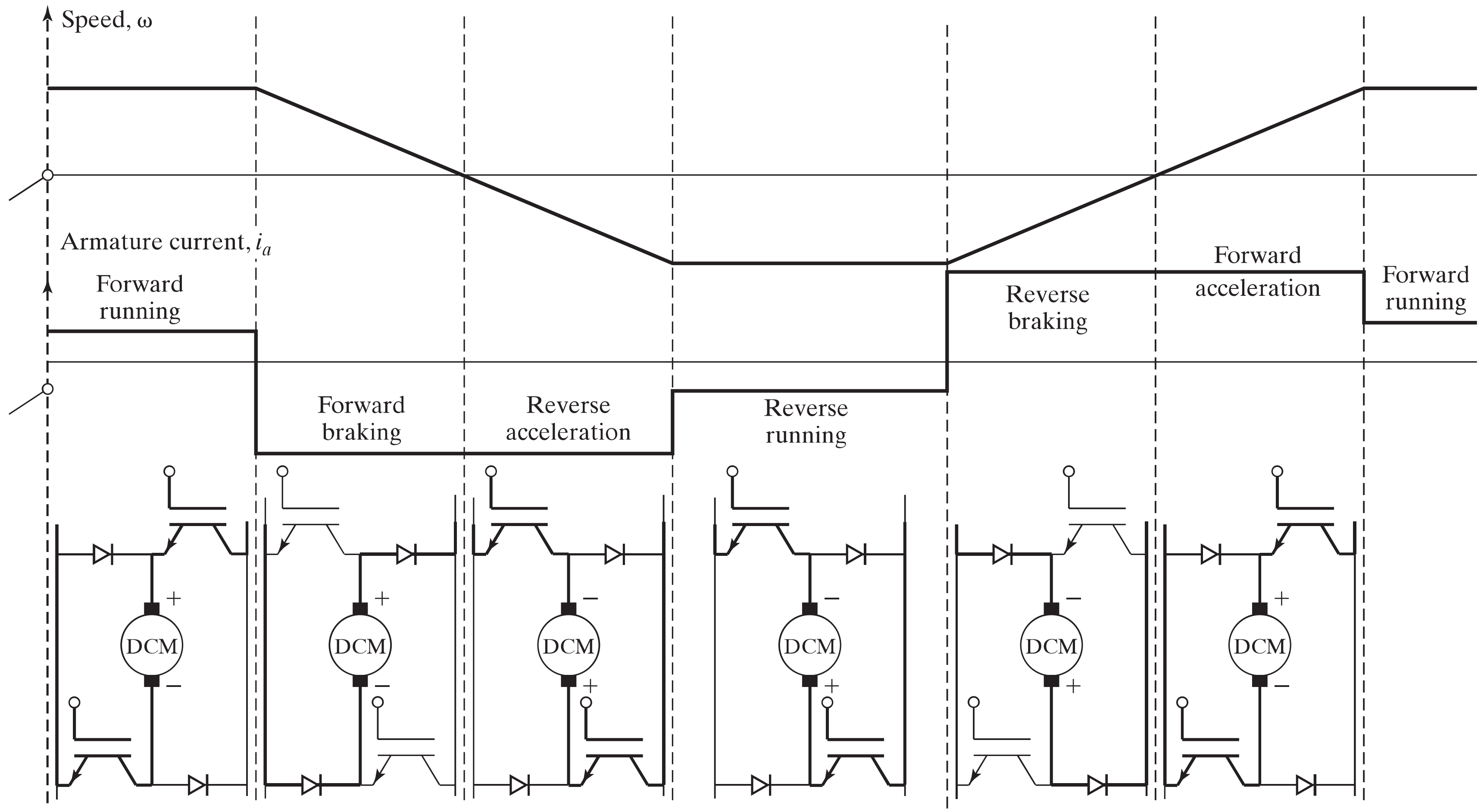


FIGURE 14.36

Typical profile of a dc–dc converter-fed four-quadrant drive.

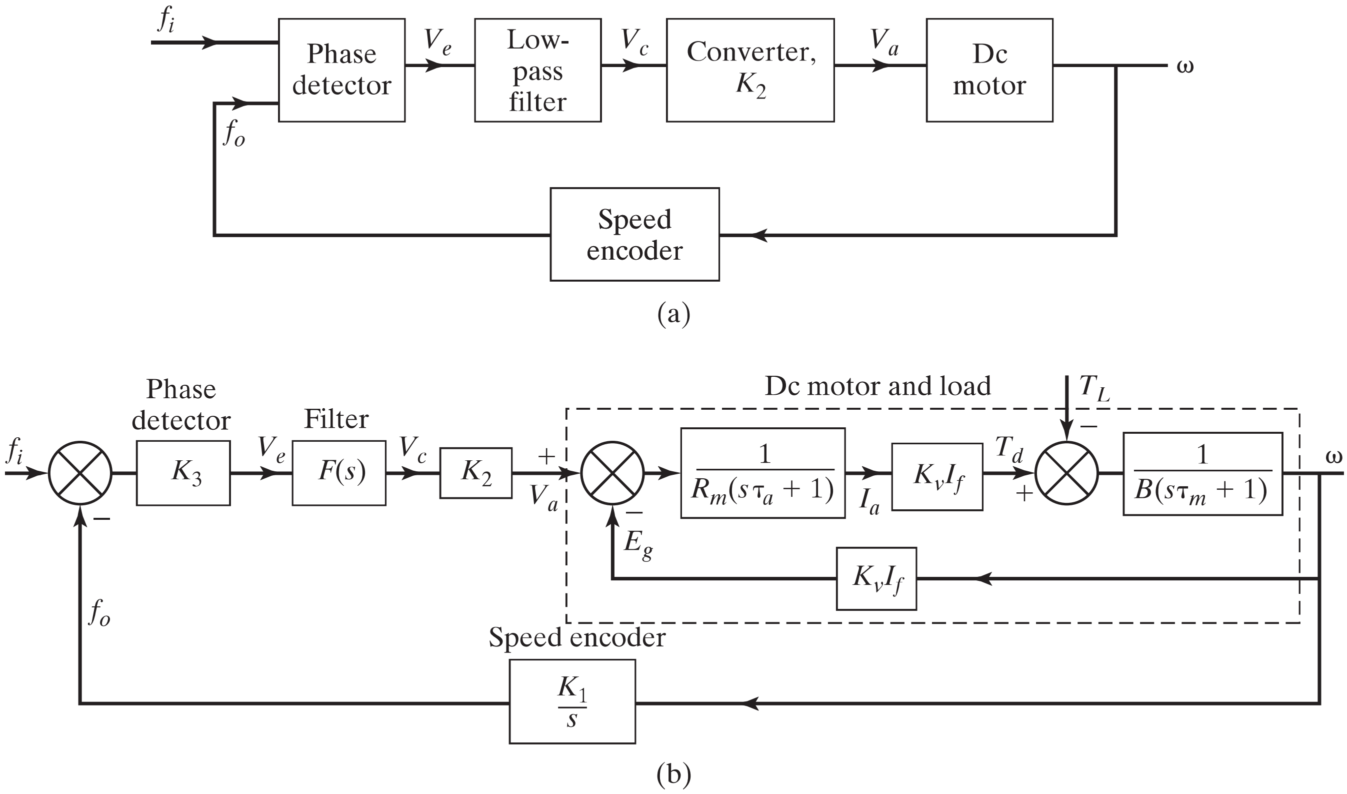


FIGURE 14.37

Phase-locked-loop control system.

would result in a phase difference and the output of the phase detector would respond immediately to vary the speed of the motor in such a direction and magnitude as to retain the locking of the reference and feedback frequencies. The response of the phase detector is very fast. As long as the two frequencies are locked, the speed regulation should ideally be zero. However, in practice the speed regulation is limited to 0.002%, and this represents a significant improvement over the analog speed control system.

14.7.11 Microcomputer Control of Dc Drives

The analog control scheme for a converter-fed dc motor drive can be implemented by hardwired electronics. An analog control scheme has several disadvantages: non-linearity of speed sensor, temperature dependency, drift, and offset. Once a control circuit is built to meet certain performance criteria, it may require major changes in the hardwired logic circuits to meet other performance requirements.

A microcomputer control reduces the size and costs of hardwired electronics, improving reliability and control performance. This control scheme is implemented in the software and is flexible to change the control strategy to meet different performance characteristics or to add extra control features. A micro-computer control system can also perform various desirable functions: on and off of the main power supply, start and stop of the drive, speed control, current control, monitoring the control variables, initiating protection and trip circuit, diagnostics for built-in fault finding, and communication with a supervisory central computer. Figure 14.38 shows a schematic diagram for a microcomputer control of a converter-fed four-quadrant dc drive.

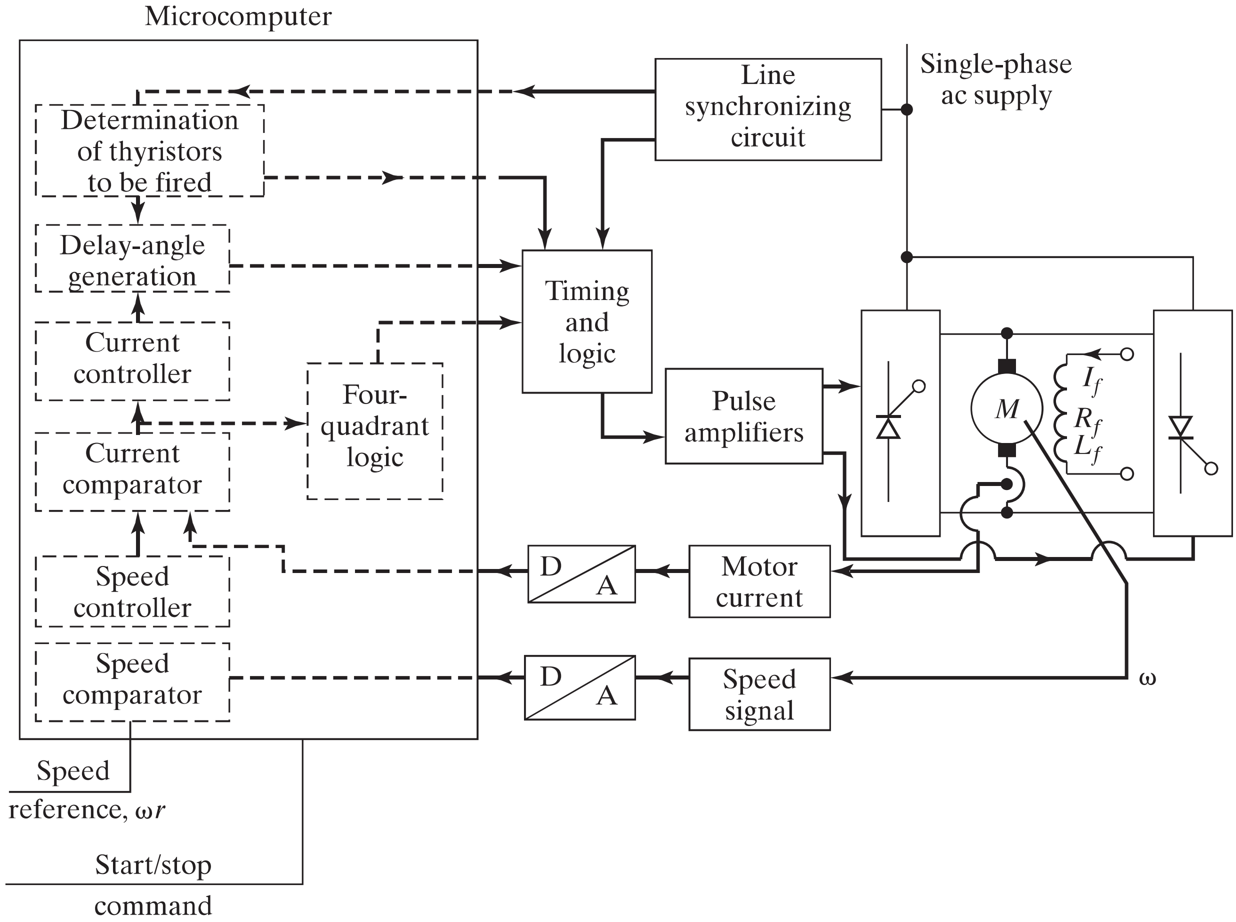


FIGURE 14.38

Schematic diagram of computer-controlled four-quadrant dc drive.

The speed signal is fed into the microcomputer using an analog-to-digital (A/D) converter. To limit the armature current of the motor, an inner current-control loop is used. The armature current signal can be fed into the microcomputer through an A/D converter or by sampling the armature current. The line synchronizing circuit is required to synchronize the generation of the firing pulses with the supply line frequency. Although the microcomputer can perform the functions of gate pulse generator and logic circuit, these are shown outside the microcomputer. The pulse amplifier provides the necessary isolation and produces gate pulses of required magnitude and duration. A microprocessor-controlled drive has become a norm. Analog control has become almost obsolete.

Key Points of Section 14.7

- The transfer function describes the speed response for any changes in torque or reference signal.
- A motor drive is required to operate at a desired speed at a certain load torque.
- An additional current loop is applied to the feedback path to provide faster response to any disturbances in speed command, load torque, and supply voltage.

SUMMARY

In dc drives, the armature and field voltages of dc motors are varied by either ac–dc converters or dc–dc converters. The ac–dc converter-fed drives are normally used in variable-speed applications, whereas dc–dc converter-fed drives are more suited for traction applications. Dc series motors are mostly used in traction applications, due to their capability of high-starting torque.

Dc drives can be classified broadly into three types depending on the input supply: (1) single-phase drives, (2) three-phase drives, and (3) dc–dc converter drives. Again each drive could be subdivided into three types depending on the modes of operation: (a) one-quadrant drives, (b) two-quadrant drives, and (c) four-quadrant drives. The energy-saving feature of dc–dc converter-fed drives is very attractive for use in transportation systems requiring frequent stops.

Closed-loop control, which has many advantages, is normally used in industrial drives. The speed regulation of dc drives can be significantly improved by using PLL control. The analog control schemes, which are hardwired electronics, are limited in flexibility and have certain disadvantages, whereas microcomputer control drives, which are implemented in software, are more flexible and can perform many desirable functions.

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REVIEW QUESTIONS

14.1 What are the three types of dc drives based on the input supply?
14.2 What is the magnetization characteristic of dc motors?
14.3 What is the purpose of a converter in dc drives?
14.4 What is a base speed of dc motors?
14.5 What are the parameters to be varied for speed control of separately excited dc motors?
14.6 Which are the parameters to be varied for speed control of dc series motors?
14.7 Why are the dc series motors mostly used in traction applications?
14.8 What is a speed regulation of dc drives?
14.9 What is the principle of single-phase full-converter-fed dc motor drives?
14.10 What is the principle of three-phase semiconverter-fed dc motor drives?
14.11 What are the advantages and disadvantages of single-phase full-converter-fed dc motor drives?
14.12 What are the advantages and disadvantages of single-phase semiconverter-fed dc motor drives?
14.13 What are the advantages and disadvantages of three-phase full-converter-fed dc motor drives?
14.14 What are the advantages and disadvantages of three-phase semiconverter-fed dc motor drives?
14.15 What are the advantages and disadvantages of three-phase dual-converter-fed dc motor drives?
14.16 Why is it preferable to use a full converter for field control of separately excited motors?
14.17 What is a one-quadrant dc drive?
14.18 What is a two-quadrant dc drive?
14.19 What is a four-quadrant dc drive?
14.20 What is the principle of regenerative braking of dc–dc converter-fed dc motor drives?
14.21 What is the principle of rheostatic braking of dc–dc converter-fed dc motor drives?
14.22 What are the advantages of dc–dc converter-fed dc drives?
14.23 What are the advantages of multiphase dc–dc converters?
14.24 What is the principle of closed-loop control of dc drives?
14.25 What are the advantages of closed-loop control of dc drives?
14.26 What is the principle of phase-locked-loop control of dc drives?
14.27 What are the advantages of phase-locked-loop control of dc drives?
14.28 What is the principle of microcomputer control of dc drives?
14.29 What are the advantages of microcomputer control of dc drives?
14.30 What is a mechanical time constant of dc motors?
14.31 What is an electrical time constant of dc motors?
14.32 Why is a cosine function normally used to generate the delay angle of controlled rectifiers?
14.33 Why is the transfer function between the speed and the reference voltage of dc motors split into two transfer functions?
14.34 What assumptions are normally made to simplify the design of the inner current-loop controller?
14.35 What assumptions are normally made to simplify the design of the outer speed-loop controller?
14.36 What are the optimum conditions for the gain and time constants of the speed controller?
14.37 What is the purpose of adding a compensating network with a pole-zero in the speed feedback path?

PROBLEMS

14.1 A separately excited dc motor is supplied from a dc source of 600 V to control the speed of a mechanical load and the field current is maintained constant. The armature resistance and losses are negligible. **(a)** If the armature current is $I_a = 145$ A at 1500 rpm, determine the load torque. **(b)** If the armature current remains the same as that in (a) and the field current is reduced such that the motor runs at a speed of 3000, determine the load torque.

14.2 Repeat Problem 14.1 if the armature resistance is $R_a = 0.10\ \Omega$. The viscous friction and the no-load losses are negligible.

14.3 A 30-hp, 440-V, 2000-rpm separately excited dc motor controls a load requiring a torque of $T_L = 85$ N·m at 1200 rpm. The back emf is $E_g = 120$ V, the armature circuit resistance is $R_a = 0.04\ \Omega$, and the motor voltage constant is $K_v = 0.7032$ V/A rad/s. The field voltage is $V_f = 440$ V. The viscous friction and the no-load losses are negligible. The armature current may be assumed continuous and ripple free. Determine **(a)** the field circuit resistance, **(b)** the required armature voltage V_a, **(c)** the rated armature current of the motor, and **(d)** the speed regulation at full load.

14.4 A 120-hp, 600-V, 1200-rpm dc series motor controls a load requiring a torque of $T_L = 185$ N·m at 1100 rpm. The field circuit resistance is $R_f = 0.06\ \Omega$, the armature circuit resistance is $R_a = 0.02\ \Omega$, and the voltage constant is $K_v = 32$ mV/A rad/s. The viscous friction and the no-load losses are negligible. The armature current is continuous and ripple free. Determine **(a)** the back emf E_g, **(b)** the required armature voltage V_a, **(c)** the rated armature current, and **(d)** the speed regulation at full speed.

14.5 The speed of a separately excited motor is controlled by a single-phase semiconverter in Figure 14.12a. The field current is also controlled by a semiconverter and the field current is set to the maximum possible value. The ac supply voltage to the armature and field converter is one phase, 208 V, 60 Hz. The armature resistance is $R_a = 0.11\ \Omega$, the field current is $I_f = 0.9$ A and the motor voltage constant is $K_v = 1.055$ V/A rad/s. The load torque is $T_L = 75$ N·m at a speed of 700 rpm. The viscous friction and no-load losses are negligible. The armature and field currents are continuous and ripple free. Determine **(a)** the field resistance R_f; **(b)** the delay angle of the converter in the armature circuit α_a; and **(c)** the input power factor (PF) of the armature circuit.

14.6 The speed of a separately excited dc motor is controlled by a single-phase full-wave converter in Figure 14.13a. The field circuit is also controlled by a full converter and the field current is set to the maximum possible value. The ac supply voltage to the armature and field converters is one phase, 208 V, 60 Hz. The armature resistance is $R_a = 0.4\ \Omega$, the field circuit resistance is $R_f = 345\ \Omega$, and the motor voltage constant is $K_v = 0.71$ V/A rad/s. The viscous friction and no-load losses are negligible. The armature and field currents are continuous and ripple free. If the delay angle of the armature converter is $\alpha_a = 45°$ and the torque developed by the motor is $T_d = 21$ N·m, determine **(a)** the armature current of the motor I_a, **(b)** the speed ω, and **(c)** the input PF of the drive.

14.7 If the polarity of the motor back emf in Problem 14.6 is reversed by reversing the polarity of the field current, determine **(a)** the delay angle of the armature circuit converter α_a to maintain the armature current constant at the same value, and **(b)** the power fed back to the supply during regenerative braking of the motor.

14.8 The speed of a 20-hp, 300-V, 1800-rpm separately excited dc motor is controlled by a three-phase full-converter drive. The field current is also controlled by a three-phase full-converter and is set to the maximum possible value. The ac input is three-phase, Y-connected, 208 V, 60 Hz. The armature resistance is $R_a = 0.25\ \Omega$, the field resistance is $R_f = 250\ \Omega$, and the motor voltage constant is $K_v = 1.15$ V/A rad/s. The armature and field currents are continuous and ripple free. The viscous friction and no-load losses are negligible. Determine **(a)** the delay angle of the armature converter α_a, if the motor supplies the rated power at the rated speed; **(b)** the no-load speed if the delay angles are the same as in (a) and the armature current at no-load is 10% of the rated value; and **(c)** the speed regulation.

14.9 Repeat Problem 14.8 if both armature and field circuits are controlled by three-phase semiconverters.

14.10 The speed of a 20-hp, 300-V, 900-rpm separately excited dc motor is controlled by a three-phase full converter. The field circuit is also controlled by a three-phase full converter. The ac input to armature and field converters is three-phase, Y-connected, 208 V, 60 Hz. The armature resistance $R_a = 0.12\ \Omega$, the field circuit resistance $R_f = 145\ \Omega$, and the motor voltage constant $K_v = 1.15$ V/A rad/s. The viscous friction and no-load losses are negligible. The armature and field currents are continuous and ripple free. **(a)** If the field converter is operated at the maximum field current and the developed torque is $T_d = 106$ N·m at 750 rpm, determine the delay angle of the armature converter α_a. **(b)** If the field circuit converter is set to the maximum field current, the developed torque is $T_d = 108$ N·m, and the delay angle of the armature converter is $\alpha_a = 0$, determine the speed. **(c)** For the same load demand as in (b), determine the delay angle of the field circuit converter if the speed has to be increased to 1800 rpm.

14.11 Repeat Problem 14.10 if both the armature and field circuits are controlled by three-phase semiconverters.

14.12 A dc–dc converter controls the speed of a dc series motor. The armature resistance $R_a = 0.06\ \Omega$, field circuit resistance $R_f = 0.04\ \Omega$, and back emf constant $K_v = 25$ mV/rad/s. The dc input voltage of the dc–dc converter $V_s = 600$ V. If it is required to maintain a constant developed torque of $T_d = 500$ N·m, plot the motor speed against the duty cycle k of the dc–dc converter.

14.13 A dc–dc converter controls the speed of a separately excited motor. The armature resistance is $R_a = 0.05\ \Omega$. The back emf constant is $K_v = 1.6$ V/A rad/s. The rated field current is $I_f = 2$ A. The dc input voltage to the dc–dc converter is $V_s = 600$ V. If it is required to maintain a constant developed torque of $T_d = 500$ N·m, plot the motor speed against the duty cycle k of the dc–dc converter.

14.14 A dc series motor is powered by a dc–dc converter, as shown in Figure 14.18a, from a 600-V dc source. The armature resistance is $R_a = 0.04\ \Omega$ and the field resistance is $R_f = 0.05\ \Omega$. The back emf constant of the motor is $K_v = 15.27$ mV/A rad/s. The average armature current $I_a = 400$ A. The armature current is continuous and has negligible ripple. If the input power from the source is $P_i = 192$ kW determine **(a)** the duty cycle of the dc–dc converter k, **(b)** the equivalent input resistance of the dc–dc converter drive, **(c)** the motor speed, and **(d)** the developed torque of the motor.

14.15 The drive in Figure 14.16a is operated in regenerative braking of a dc series motor. The dc supply voltage is 600 V. The armature resistance is $R_a = 0.04\ \Omega$ and the field resistance is $R_f = 0.06\ \Omega$. The back emf constant of the motor is $K_v = 12$ mV/A rad/s. The average armature current is maintained constant at $I_a = 350$ A. The armature current is continuous and has negligible ripple. If the average voltage across the dc–dc converter is $V_{ch} = 300$ V, determine **(a)** the duty cycle of the dc–dc converter; **(b)** the power regenerated to the dc supply P_g; **(c)** the equivalent load resistance of the motor acting as a generator, R_{eq}; **(d)** the minimum permissible braking speed ω_{min}; **(e)** the maximum permissible braking speed ω_{max}; and **(f)** the motor speed.

14.16 A dc–dc converter is used in rheostatic braking of a dc series motor, as shown in Figure 14.17. The armature resistance $R_a = 0.04\ \Omega$ and the field resistance $R_f = 0.04\ \Omega$. The braking resistor $R_b = 5\ \Omega$. The back emf constant $K_v = 14$ mV/A rad/s. The average armature current is maintained constant at $I_a = 200$ A. The armature current is continuous and has negligible ripple. If the duty cycle of the dc–dc converter is 60%, determine **(a)** the average voltage across the dc–dc converter V_{ch}; **(b)** the power dissipated in the resistor P_b; **(c)** the equivalent load resistance of the motor acting as a generator, R_{eq}; **(d)** the motor speed; and **(e)** the peak dc–dc converter voltage V_p.

14.17 Two dc–dc converters control a dc motor, as shown in Figure 14.21a, and they are phase shifted in operation by π/m, where m is the number of multiphase dc–dc converters. The supply voltage $V_s = 440$ V, total armature circuit resistance $R_m = 6.5\ \Omega$, armature circuit inductance $L_m = 12$ mH, and the frequency of each dc–dc converter $f = 250$ Hz. Calculate the maximum value of peak-to-peak load ripple current.

14.18 For Problem 14.17, plot the maximum value of peak-to-peak load ripple current against the number of multiphase dc–dc converters.

14.19 A dc motor is controlled by two multiphase dc–dc converters. The average armature current is $I_a = 300$ A. A simple LC-input filter with $L_e = 0.35$ mH and $C_e = 5600\ \mu$F is used. The rms fundamental component of the dc–dc converter-generated harmonic current in the supply is $I_{s1} = 6$ A. Determine the frequency f at which each dc–dc converter is operated.

14.20 For Problem 14.19, plot the rms fundamental component of the dc–dc converter-generated harmonic current in the supply against the number of multiphase dc–dc converters.

14.21 A 40-hp, 230-V, 3500-rpm separately excited dc motor is controlled by a linear converter of gain $K_2 = 200$. The moment of inertia of the motor load is $J = 0.156$ N·m/rad/s, viscous friction constant is negligible, total armature resistance is $R_m = 0.045\ \Omega$, and total armature inductance is $L_m = 730$ mH. The back emf constant is $K_v = 0.502$ V/A rad/s and the field current is maintained constant at $I_f = 1.25$ A. **(a)** Obtain the open-loop transfer function $\omega(s)/V_r(s)$ and $\omega(s)/T_L(s)$ for the motor. **(b)** Calculate the motor steady-state speed if the reference voltage is $V_r = 1$ V and the load torque is 60% of the rated value.

14.22 Repeat Problem 14.21 with a closed-loop control if the amplification of speed sensor is $K_1 = 3$ mV/rad/s.

14.23 The motor in Problem 14.21 is controlled by a linear converter of gain K_2 with a closed-loop control. If the amplification of speed sensor is $K_1 = 3$ mV/rad/s, determine the gain of the converter K_2 to limit the speed regulation at full load to 1%.

14.24 A 60-hp, 230-V, 1750-rpm separately excited dc motor is controlled by a converter, as shown in the block diagram in Figure 14.29. The field current is maintained constant at $I_f = 1.25$ A and the machine back emf constant is $K_v = 0.81$ V/A rad/s. The armature resistance is $R_a = 0.02\ \Omega$ and the viscous friction constant is $B = 0.3$ N·m/rad/s. The amplification of the speed sensor is $K_1 = 96$ mV/rad/s and the gain of the power controller is $K_2 = 150$. **(a)** Determine the rated torque of the motor. **(b)** Determine the reference voltage V_r to drive the motor at the rated speed. **(c)** If the reference voltage is kept unchanged, determine the speed at which the motor develops the rated torque.

14.25 Repeat Problem 14.24. **(a)** If the load torque is increased by 20% of the rated value, determine the motor speed. **(b)** If the reference voltage is reduced by 10%, determine the motor speed. **(c)** If the load torque is reduced by 15% of the rated value and the reference voltage is reduced by 20%, determine the motor speed. **(d)** If there was no feedback, as in an open-loop control, determine the speed regulation for a reference voltage $V_r = 1.24$ V. **(e)** Determine the speed regulation with a closed-loop control.

14.26 A 40-hp, 230-V, 3500-rpm series excited dc motor is controlled by a linear converter of gain $K_2 = 200$. The moment of inertia of the motor load $J = 0.156$ N·m/rad/s, viscous friction constant is negligible, total armature resistance is $R_m = 0.045\ \Omega$, and total armature inductance is $L_m = 730$ mH. The back emf constant is $K_v = 340$ mV/A rad/s. The field resistance is $R_f = 0.035\ \Omega$ and field inductance is $L_f = 450$ mH. **(a)** Obtain the open-loop transfer function $\omega(s)/V_r(s)$ and $\omega(s)/T_L(s)$ for the motor. **(b)** Calculate the motor steady-state speed if the reference voltage, $V_r = 1$ V, and the load torque is 60% of the rated value.

14.27 Repeat Problem 14.26 with closed-loop control if the amplification of speed sensor $K_1 = 3$ mV/rad/s.

14.28 The parameters of a separately excited dc motor drive are $R_m = 0.6\ \Omega$, $L_m = 3.5$ mH, $K_m = 0.51$ V/rad/s, $J = 0.0177$ kg-m^2, $B = 0.02$ Nm/rad/s, and a speed of 1800 rpm. The armature dc supply voltage is 220 V. If the motor is operated at its rated field current $I_f = 1.5$ A, determine the load torque developed by the motor.

14.29 The parameters of the gearbox shown in Figure 14.7 are $B_1 = 0.035$ Nm/rad/s, $\omega_1 = 300$ rad/s, $B_m = 0.064$ kg-m^2, $J_m = 0.35$ kg-m^2, $J_1 = 0.25$ kg-m^2, $T_2 = 25$ Nm, the gear ratio $GR = 15$. Determine **(a)** the ω_2, **(b)** the effective motor torque T_1, **(c)** the effective inertia J, and **(d)** the effective friction coefficient B.

14.30 The parameters of the gearbox shown in Figure 14.7 are $B_1 = 0.034$ Nm/rad/s, $\omega_1 = 490$ rad/s, $B_m = 0.064$ kg-m^2, $J_m = 0.35$ kg-m^2, $J_1 = 0.25$ kg-m^2, the effective motor torque $T_1 = 0.2$ Nm, and $\omega_2 = 35$ rad/s. Determine **(a)** the gear ratio $GR = N_1/N_2$ **(b)** the motor torque T_2, **(c)** the effective inertia J, and **(d)** the effective friction coefficient B.

14.31 Repeat Example 14.14 if the maximum permissible motor current is 40 A and the ac supply frequency is $f_s = 50$ Hs.

14.32 Repeat Example 14.14 if the converter is supplied from a single-phase 120 V ac source at 60 Hz.

14.33 Repeat Example 14.14 if the converter is supplied from a single-phase 120 V ac source at 50 Hz.

14.34 Repeat Example 14.14 if the converter is supplied from a single-phase 240 V ac source at 50 Hz.

14.35 For Example 14.14, (a) plot the transient response $\omega_r(t)$ in Eq. (14.124) from 0 to 100 ms, and (b) the rise time, the maximum overshoot, and the settling time.

14.36 For Example 14.14, (a) plot the transient response $\omega_r(t)$ in Eq. (14.126) from 0 to 100 ms, and (b) the rise time, the maximum overshoot, and the settling time.

CHAPTER 15

Ac Drives

After completing this chapter, students should be able to do the following:

- Describe the speed–torque characteristics of induction motors.
- List the methods for speed control of induction motors.
- Determine the performance parameters of induction motors.
- Explain the principle of vector or field-oriented control for induction motors.
- List the types of synchronous motors.
- Determine the performance parameters of synchronous motors.
- Describe the control characteristics of synchronous motors and the methods for speed control.
- Explain the methods for speed control of stepper motors.
- Explain the operation of linear induction motors.
- Determine the performance parameters of linear induction motors.

Symbols and Their Meanings

Symbols	Meaning
E_r; E_m	Rms and peak rotor induced voltage per phase, respectively
f; V_{dc}	Supply frequency and dc supply voltage, respectively
$f_{\alpha s}$; $f_{\beta s}$; f_o	Stator variables in the α-β frame, respectively
f_{ds}; f_{qs}; f_o	Stator variables in the d-q frame, respectively
i_{qs}; i_{ds}; i_{qr}; i_{dr}	Stator and rotor current in the q-d synchronous frame, respectively
I_s; I_r	Rms stator and rotor currents in their self-windings, respectively
P_i; P_g; P_d; P_o	Input, gap, developed and output powers, respectively
R_s; X_s	Stator per phase resistance and reactance in the rotor windings, respectively
R'_r; X'_r	Rotor per phase resistance and reactance reflected in the stator windings, respectively
R_m; X_m	Magnetizing resistance and reactance, respectively
s; s_m	Slip and slip or maximum toque, respectively
T_L; T_e; T_s; T_d; T_m; T_{mm}	Load, electromagnetic, starting, developed, maximum and breakdown torques, respectively
v_{qs}; v_{ds}; v_{qr}; v_{dr}	Stator and rotor voltages in the q-d synchronous frame, respectively
V_m; $i_{as}(t)$	Peak and instantaneous stator current, respectively

(*continued*)

Symbols	Meaning
V_s; V_a	Rms supply and rms applied voltages, respectively
α; δ	Delay and torque angles, respectively
β; b	Frequency ratio and voltage-to-frequency ratios, respectively
ω_s; ω; ω_m; ω_b; ω_{sl}	Synchronous, supply, motor, base and slip speeds, respectively in rad/s
Φ; K_m; b	Flux, motor constant and voltage ratio, respectively

15.1 INTRODUCTION

Ac motors exhibit highly coupled, nonlinear, and multivariable structures as opposed to much simpler decoupled structures of separately excited dc motors. The control of ac drives generally requires complex control algorithms that can be performed by microprocessors or microcomputers along with fast-switching power converters.

The ac motors have a number of advantages; they are lightweight (20% to 40% lighter than equivalent dc motors), are inexpensive, and have low maintenance compared with dc motors. They require control of frequency, voltage, and current for variable-speed applications. The power converters, inverters, and ac voltage controllers can control the frequency, voltage, or current to meet the drive requirements. These power controllers, which are relatively complex and more expensive, require advanced feedback control techniques such as model reference, adaptive control, sliding mode control, and field-oriented control. However, the advantages of ac drives outweigh the disadvantages. There are four types of ac drives:

1. Induction motor drives
2. Synchronous motor drives
3. Stepper motor drives
4. Linear induction motor

Ac drives are replacing dc drives and are used in many industrial and domestic applications [1, 2].

15.2 INDUCTION MOTOR DRIVES

Three-phase induction motors are commonly used in adjustable-speed drives [1] and they have three-phase stator and rotor windings. The stator windings are supplied with balanced three-phase ac voltages, which produce induced voltages in the rotor windings due to transformer action. It is possible to arrange the distribution of stator windings so that there is an effect of multiple poles, producing several cycles of magnetomotive force (mmf) (or field) around the air gap. This field establishes a spatially distributed sinusoidal flux density in the air gap. The speed of rotation of the field is called the *synchronous speed*, which is defined by

$$\omega_s = \frac{2\omega}{p} \tag{15.1}$$

where p is the number of poles and ω is the supply frequency in rads per second.

If a stator phase voltage, $v_s = \sqrt{2}V_s \sin \omega t$, produces a flux linkage (in the rotor) given by

$$\phi(t) = \phi_m \cos(\omega_m t + \delta - \omega_s t) \tag{15.2}$$

the induced voltage per phase in the rotor winding is

$$\begin{aligned} e_r &= N_r \frac{d\phi}{dt} = N_r \frac{d}{dt}[\phi_m \cos(\omega_m t + \delta - \omega_s t)] \\ &= -N_r \phi_m (\omega_s - \omega_m) \sin[(\omega_s - \omega_m)t - \delta] \\ &= -sE_m \sin(s\omega_s t - \delta) \\ &= -s\sqrt{2}E_r \sin(s\omega_s t - \delta) \end{aligned} \tag{15.3}$$

where N_r = number of turns on each rotor phase;
ω_m = angular rotor speed or frequency, Hz;
δ = relative position of the rotor;
E_r = rms value of the induced voltage in the rotor per phase, V;
E_m = peak induced voltage in the rotor per phase, V.

and s is the slip, defined as

$$s = \frac{\omega_s - \omega_m}{\omega_s} \tag{15.4}$$

which gives the motor speed as $\omega_m = \omega_s(1 - s)$. ω_s can be considered as the maximum mechanical speed ω_{rm} that corresponds to the supply frequency (or speed) ω and the slip speed becomes $\omega_{sl} = \omega_{rm} - \omega_m = \omega_s - \omega_m$. It is also possible to convert a mechanical speed ω_m to the rotor electrical speed ω_{re} of the rotating field as given by

$$\omega_{re} = \frac{p}{2}\omega_m \tag{15.4a}$$

In that case, ω is the synchronous electrical speed, ω_{re}. The slip speed becomes $\omega_{sl} = \omega - \omega_{re} = \omega - \omega_r$. Thus, the slip can also defined as

$$s = \frac{\omega - \omega_r}{\omega} = 1 - \frac{\omega_r}{\omega} \tag{15.4b}$$

Which gives the rotor electrical speed as

$$\omega_r = \omega(1 - s) \tag{15.4c}$$

This relates directly to the supply frequency ω and is often convenient for analyzing inductor motor drives (Section 15.5.2). The motor speed is often set to a desired value and the rotor speed is added to the slip speed to calculate the desired the supply frequency. The supply frequency and the slip are varied to control the motor speed (Section 16.3). The equivalent circuit for one phase of the rotor is shown in Figure 15.1a,

where R_r is the resistance per phase of the rotor windings;
X_r is the leakage reactance per phase of the rotor at the supply frequency;
E_r represents the induced rms phase voltage when the speed is zero (or $s = 1$).

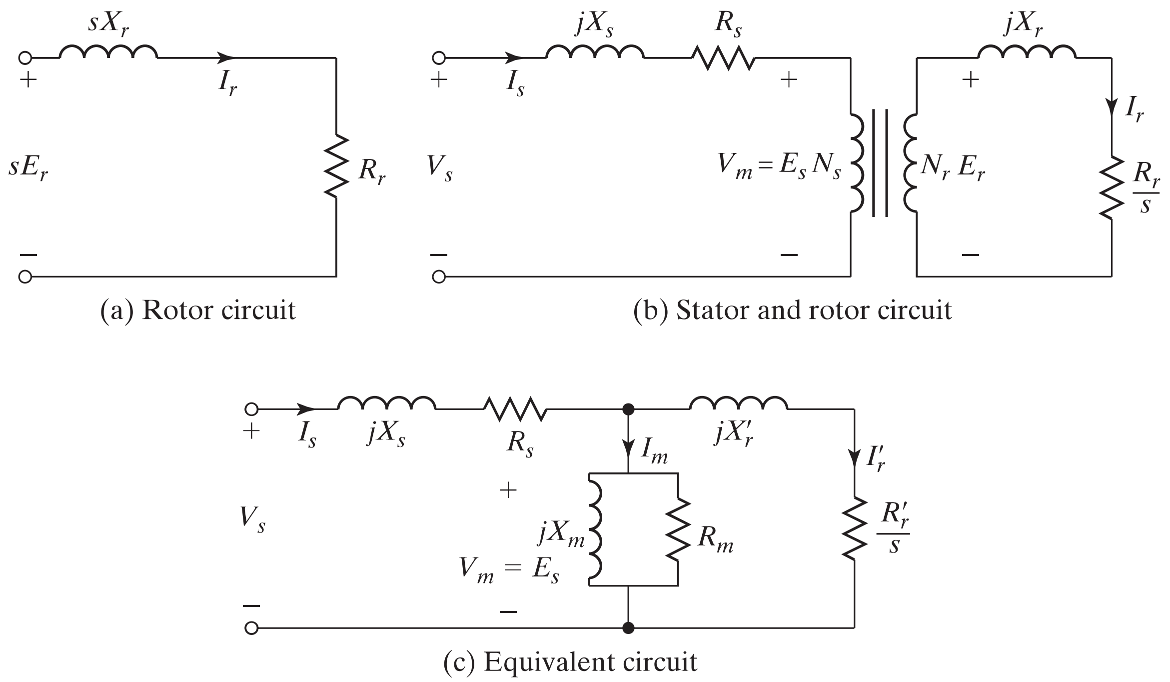


FIGURE 15.1

Circuit model of induction motors.

The rotor current is given by

$$I_r = \frac{sE_r}{R_r + jsX_r} \tag{15.5}$$

$$= \frac{E_r}{R/s + jX_r} \tag{15.5a}$$

where R_r and X_r are referred to the rotor winding.

The per-phase circuit model of induction motors is shown in Figure 15.1b, where R_s and X_s are the per-phase resistance and leakage reactance of the stator winding. The complete circuit model with all parameters referred to the stator is shown in Figure 15.1c, where R_m represents the resistance for excitation (or core) loss and X_m is the magnetizing reactance. R_r' and X_r' are the rotor resistance and reactance referred to the stator. I_r' is the rotor current referred to the stator. There will be stator core loss, when the supply is connected and the rotor core loss depends on the slip. The friction and windage loss $P_{\text{no load}}$ exists when the machine rotates. The core loss P_c may be included as a part of rotational loss $P_{\text{no load}}$.

15.2.1 Performance Characteristics

The rotor current I_r and stator current I_s can be found from the circuit model in Figure 15.1c where R_r and X_r are referred to the stator windings. Once the values of I_r and I_s are known, the performance parameters of a three-phase motor can be determined as follows:

Stator copper loss

$$P_{su} = 3 I_s^2 R_s \tag{15.6}$$

Rotor copper loss

$$P_{ru} = 3(I_r')^2 R_r' \tag{15.7}$$

Core loss

$$P_c = \frac{3V_m^2}{R_m} \approx \frac{3V_s^2}{R_m} \tag{15.8}$$

Gap power (power passing from the stator to the rotor through the air gap)

$$P_g = 3(I_r')^2 \frac{R_r'}{s} \tag{15.9}$$

Developed power

$$P_d = P_g - P_{ru} = 3(I_r')^2 \frac{R_r'}{s}(1-s) \tag{15.10}$$

$$= P_g(1-s) \tag{15.11}$$

Developed torque

$$T_d = \frac{P_d}{\omega_m} \tag{15.12}$$

$$= \frac{P_g(1-s)}{\omega_s(1-s)} = \frac{P_g}{\omega_s} \tag{15.12a}$$

Input power

$$P_i = 3\,V_s I_s \cos\theta_m \tag{15.13}$$

$$= P_c + P_{su} + P_g \tag{15.13a}$$

where θ_m is the angle between I_s and V_s. Output power

$$P_o = P_d - P_{\text{no load}}$$

Efficiency

$$\eta = \frac{P_o}{P_i} = \frac{P_d - P_{\text{no load}}}{P_c + P_{su} + P_g} \tag{15.14}$$

If $P_g \gg (P_c + P_{su})$ and $P_d \gg P_{\text{no load}}$, the efficiency becomes approximately

$$\eta \approx \frac{P_d}{P_g} = \frac{P_g(1-s)}{P_g} = 1 - s \tag{15.14a}$$

The value of X_m is normally large and R_m, which is much larger, can be removed from the circuit model to simplify the calculations. If $X_m^2 \gg (R_s^2 + X_s^2)$, then $V_s \approx V_m$, and the magnetizing reactance X_m may be moved to the stator winding to simplify further; this is shown in Figure 15.2.

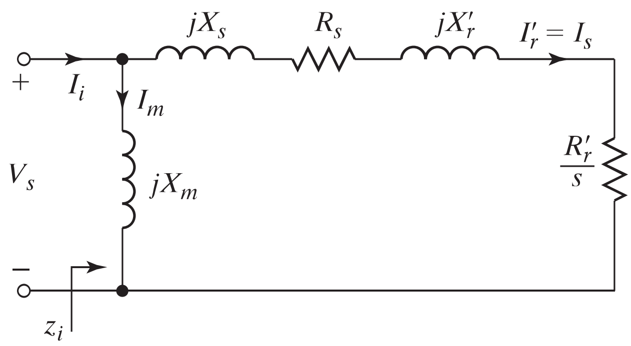


FIGURE 15.2
Approximate per-phase equivalent circuit.

The input impedance of the motor becomes

$$\mathbf{Z}_i = \frac{-X_m(X_s + X'_r) + jX_m(R_s + R'_r/s)}{R_s + R'_r/s + j(X_m + X_s + X'_r)} \tag{15.15}$$

and the power factor (PF) angle of the motor

$$\theta_m = \pi - \tan^{-1}\frac{R_s + R'_r/s}{X_s + X'_r} + \tan^{-1}\frac{X_m + X_s + X'_r}{R_s + R'_r/s} \tag{15.16}$$

From Figure 15.2, the rms rotor current

$$I'_r = \frac{V_s}{[(R_s + R'_r/s)^2 + (X_s + X'_r)^2]^{1/2}} \tag{15.17}$$

Substituting I'_r from Eq. (15.17) in Eq. (15.9) and then P_g in Eq. (15.12a) yields

$$T_d = \frac{3R'_r V_s^2}{s\omega_s[(R_s + R'_r/s)^2 + (X_s + X'_r)^2]} \tag{15.18}$$

15.2.2 Torque–Speed Characteristics

If the motor is supplied from a fixed voltage at a constant frequency, the developed torque is a function of the slip and the torque–speed characteristics can be determined from Eq. (15.18). A typical plot of developed torque as a function of slip or speed is shown in Figure 15.3. The slip is used as the variable instead of the rotor speed because it is nondimensional, and it is applicable to any motor frequency. Near the synchronous speed, that is, at low slips, the torque is linear and is proportional to slip. Beyond the maximum torque (also known as *breakdown torque*), the torque is inversely proportional to slip as shown in Figure 15.3. At standstill, the slip equals unity, and the torque produced is known as *standstill torque*. To accelerate a load, this standstill torque has to be greater than the load torque. It is desirable that the motor operate close to the low-slip range for higher efficiency. This is due to the fact that the rotor copper losses are directly proportional to slip and are equal to the slip power. Thus, at low slips, the rotor copper losses are small. The operation in the reverse motoring and regenerative braking is obtained by the reversal of the phase sequence of the motor terminals. The reverse speed–torque characteristics are shown by dashed lines. There are three regions of operation: (1) motoring or powering, $0 \le s \le 1$; (2) regeneration, $s < 0$; and (3) plugging, $1 \le s \le 2$.

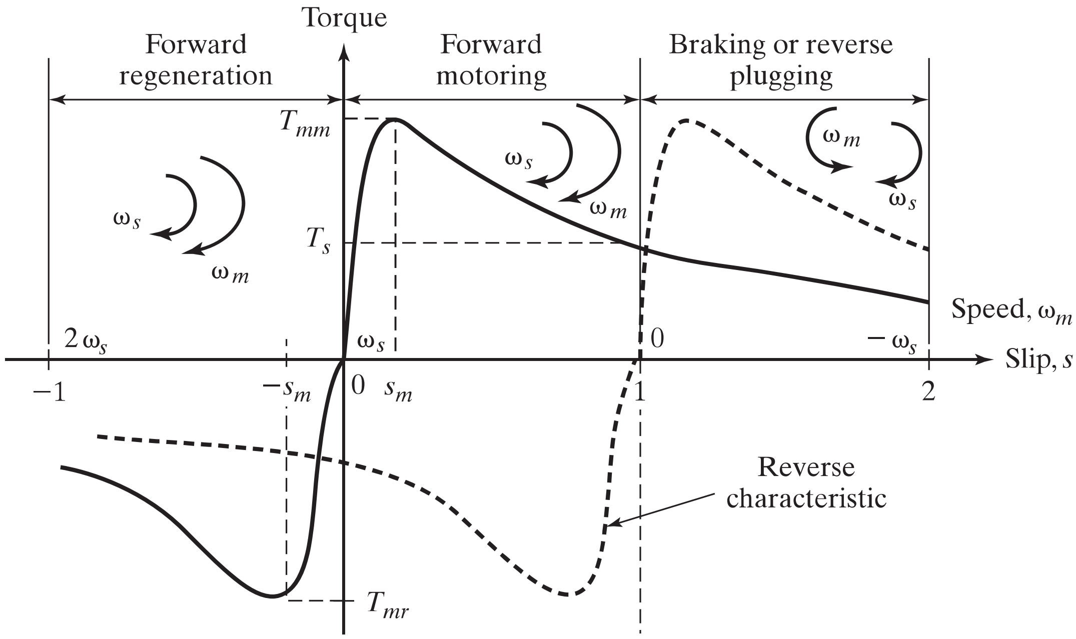


FIGURE 15.3
Torque–speed characteristics.

In motoring, the motor rotates in the same direction as the field; as the slip increases, the torque also increases while the air-gap flux remains constant. Once the torque reaches its maximum value, T_m at $s = s_m$, the torque decreases, with the increase in slip due to reduction of the air-gap flux. For a low slip such that $s < s_m$, the positive slope of the characteristic provides stable operation. If the load torque is increased, the rotor slows down and thereby develops a larger slip, which increases the electromagnetic torque capable of meeting the load torque. If the motor is operating at a slip $s > s_m$, any load torque disturbance will lead to increased slip, resulting in less and less torque generation. As a result, the developed torque will diverge more and more from the load torque demand leading to a final pullout of the machine and reaching standstill.

In regeneration, the speed ω_m is greater than the synchronous speed ω_s with ω_m and ω_s in the same direction, and the slip is negative. Therefore, R_r'/s is negative. This means that power is fed back from the shaft into the rotor circuit and the motor operates as a generator. The motor returns power to the supply system. The torque–speed characteristic is similar to that of motoring, but having negative value of torque. A negative slip causes a change in the operating mode from the generation of positive torque (motoring) to negative torque (generating) as the induced emf in phase is reversed. The regenerating breakdown torque g is much higher with negative-slip operation. This is due to the fact that the mutual flux linkages are strengthened by the generator action of the induction machine. The reversal of rotor current reduces the motor impedance voltage drop, resulting in a boost of magnetizing current and hence in an increase of mutual flux linkages and torque.

In reverse plugging, the speed is opposite to the direction of the field and the slip is greater than unity. This may happen if the sequence of the supply source is reversed while forward motoring, so that the direction of the field is also reversed. The developed torque, which is in the same direction as the field, opposes the motion and acts as braking torque. For example, if a motor is spinning in the direction opposite to that of a phase sequence (abc) and a set of stator voltages with a phase sequence (abc) is applied at supply frequency, this creates a stator flux linkage counter to the direction of rotor speed, resulting in a braking action. This also creates a slip greater than one and the rotor speed is negative with respect to synchronous speed. This braking action brings rotor speed to standstill in a short time. Because $s > 1$, the motor currents are high, but the developed torque is low. The energy due to a plugging brake must be dissipated within the motor and this may cause excessive heating of the motor. This type of braking is not normally recommended.

At starting, the machine speed is $\omega_m = 0$ and $s = 1$. The starting torque can be found from Eq. (15.18) by setting $s = 1$ as

$$T_s = \frac{3R_r'V_s^2}{\omega_s[(R_s + R_r')^2 + (X_s + X_r')^2]} \tag{15.19}$$

The slip for maximum torque s_m can be determined by setting $dT_d/ds = 0$ and Eq. (15.18) yields

$$s_m = \pm\frac{R_r'}{[R_s^2 + (X_s + X_r')^2]^{1/2}} \tag{15.20}$$

Substituting $s = s_m$ in Eq. (15.18) gives the maximum developed torque during motoring, which is also called *pull-out torque*, or *breakdown torque*,

$$T_{mm} = \frac{3V_s^2}{2\omega_s[R_s + \sqrt{R_s^2 + (X_s + X_r')^2}]} \tag{15.21}$$

and the maximum regenerative torque can be found from Eq. (15.18) by letting

$$s = -s_m$$

$$T_{mr} = \frac{3V_s^2}{2\omega_s[-R_s + \sqrt{R_s^2 + (X_s + X_r')^2}]} \tag{15.22}$$

If R_s is considered small compared with other circuit impedances, which is usually a valid approximation for motors of more than 1-kW rating, the corresponding expressions become

$$T_d = \frac{3R_r'V_s^2}{s\omega_s[(R_r'/s)^2 + (X_s + X_r')^2]} \tag{15.23}$$

$$T_s = \frac{3R_r'V_s^2}{\omega_s[(R_r')^2 + (X_s + X_r')^2]} \tag{15.24}$$

$$s_m = \pm\frac{R_r'}{X_s + X_r'} \tag{15.25}$$

$$T_{mm} = -T_{mr} = \frac{3V_s^2}{2\omega_s(X_s + X_r')} \tag{15.26}$$

Normalizing Eqs. (15.23) and (15.24) with respect to Eq. (15.26) gives

$$\frac{T_d}{T_{mm}} = \frac{2R_r'(X_s + X_r')}{s[(R_r'/s)^2 + (X_s + X_r')^2]} = \frac{2ss_m}{s_m^2 + s^2} \tag{15.27}$$

and

$$\frac{T_s}{T_{mm}} = \frac{2R_r'(X_s + X_r')}{(R_r')^2 + (X_s + X_r')^2} = \frac{2s_m}{s_m^2 + 1} \tag{15.28}$$

If $s < 1, s^2 < s_m^2$ and Eq. (15.27) can be approximated to

$$\frac{T_d}{T_{mm}} = \frac{2s}{s_m} = \frac{2(\omega_s - \omega_m)}{s_m\omega_s} \tag{15.29}$$

which gives the speed as a function of torque,

$$\omega_m = \omega_s\left(1 - \frac{s_m}{2T_{mm}}T_d\right) \tag{15.30}$$

It can be noticed from Eqs. (15.29) and (15.30) that if the motor operates with small slip, the developed torque is proportional to slip and the speed decreases with torque. The rotor current, which is zero at the synchronous speed, increases due to the decrease in R_r/s as the speed is decreased. The developed torque also increases until it becomes maximum at $s = s_m$. For $s < s_m$, the motor operates stably on the portion of the speed–torque characteristic. If the rotor resistance is low, s_m is low. That is, the change of motor speed from no-load to rated torque is only a small percentage. The motor operates essentially at a constant speed. When the load torque exceeds the breakdown torque, the motor stops and the overload protection must immediately disconnect the source to prevent damage due to overheating. It should be noted that for $s > s_m$, the torque decreases despite an increase in the rotor current and the operation is unstable for most motors. The speed and torque of induction motors can be varied by one of the following means [3–8]:

1. Stator voltage control
2. Rotor voltage control
3. Frequency control
4. Stator voltage and frequency control
5. Stator current control
6. Voltage, current, and frequency control

To meet the torque–speed duty cycle of a drive, the voltage, current, and frequency control are normally used.

Example 15.1 Finding the Performance Parameters of a Three-Phase Induction Motor

A three-phase, 460-V, 60-Hz, four-pole Y-connected induction motor has the following equivalent-circuit parameters: $R_s = 0.42\ \Omega$, $R_r' = 0.23\ \Omega$, $X_s = X_r' = 0.82\ \Omega$, and $X_m = 22\ \Omega$. The no-load loss, which is $P_{\text{no load}} = 60$ W, may be assumed constant. The rotor speed is 1750 rpm. Use the approximate equivalent circuit in Figure 15.2 to determine (a) the synchronous speed ω_s; (b) the slip s; (c) the input current I_i; (d) the input power P_i; (e) the input PF of the supply, PF_s; (f) the gap power P_g; (g) the rotor copper loss P_{ru}; (h) the stator copper loss P_{su}; (i) the developed torque T_d;

(j) the efficiency; (k) the starting current I_{rs} and starting torque T_s; (l) the slip for maximum torque s_m; (m) the maximum developed torque in motoring, T_{mm}; (n) the maximum regenerative developed torque T_{mr}; and (o) T_{mm} and T_{mr} if R_s is neglected.

Solution

$f = 60$ Hz, $p = 4$, $R_s = 0.42\ \Omega$, $R'_r = 0.23\ \Omega$, $X_s = X'_r = 0.82\ \Omega$, $X_m = 22\ \Omega$, and $N =$ 1750 rpm. The phase voltage is $V_s = 460/\sqrt{3} = 265.58$ V, $\omega = 2\pi \times 60 = 377$ rad/s, and $\omega_m = 1750\ \pi/30 = 183.26$ rad/s.

a. From Eq. (15.1), $\omega_s = 2\omega/p = 2 \times 377/4 = 188.5$ rad/s.

b. From Eq. (15.4), $s = (188.5 - 183.26)/188.5 = 0.028$.

c. From Eq. (15.15),

$$\mathbf{Z}_i = \frac{-22 \times (0.82 + 0.82) + j22 \times (0.42 + 0.23/0.028)}{0.42 + 0.23/0.028 + j(22 + 0.82 + 0.82)} = 7.732\ \underline{/30.88^\circ}$$

$$\mathbf{I}_i = \frac{V_s}{Z_i} = \frac{265.58}{7.732}\underline{/-30.8^\circ} = 34.35\underline{/-30.88^\circ}\text{A}$$

d. The PF of the motor is

$$\text{PF}_m = \cos(-30.88^\circ) = 0.858\ (\text{lagging})$$

From Eq. (15.13),

$$P_i = 3 \times 265.58 \times 34.35 \times 0.858 = 23{,}482\ \text{W}$$

e. The PF of the input supply is $\text{PF}_s = \text{PF}_m = 0.858$ (lagging), which is the same as the motor PF, PF_m, because the supply is sinusoidal.

f. From Eq. (15.17), the rms rotor current is

$$I'_r = \frac{265.58}{[(0.42 + 0.23/0.028)^2 + (0.82 + 0.82)^2]^{1/2}} = 30.1\ \text{A}$$

From Eq. (15.9),

$$P_g = \frac{3 \times 30.1^2 \times 0.23}{0.028} = 22{,}327\ \text{W}$$

g. From Eq. (15.7), $P_{ru} = 3 \times 30.1^2 \times 0.23 = 625$ W.

h. The stator copper loss is $P_{su} = 3 \times 30.1^2 \times 0.42 = 1142$ W.

i. From Eq. (15.12a), $T_d = 22{,}327/188.5 = 118.4$ N·m.

j. $P_0 = P_g - P_{ru} - P_{\text{no load}} = 22{,}327 - 625 - 60 = 21{,}642$ W.

k. For $s = 1$, Eq. (15.17) gives the starting rms rotor current

$$I_{rs} = \frac{265.58}{[(0.42 + 0.23)^2 + (0.82 + 0.82)^2]^{1/2}} = 150.5\ \text{A}$$

From Eq. (15.19),

$$T_s = \frac{3 \times 0.23 \times 150.5^2}{188.5} = 82.9\ \text{N} \cdot \text{m}$$

l. From Eq. (15.20), the slip for maximum torque (or power)

$$s_m = \pm\frac{0.23}{[0.42^2 + (0.82 + 0.82)^2]^{1/2}} = \pm 0.1359$$

m. From Eq. (15.21), the maximum developed torque

$$T_{mm} = \frac{3 \times 265.58^2}{2 \times 188.5 \times [0.42 + \sqrt{0.42^2 + (0.82 + 0.82)^2}]}$$
$$= 265.64 \text{ N} \cdot \text{m}$$

n. From Eq. (15.22), the maximum regenerative torque is

$$T_{mr} = -\frac{3 \times 265.58^2}{2 \times 188.5 \times [-0.42 + \sqrt{0.42^2 + (0.82 + 0.82)^2}]}$$
$$= -440.94 \text{ N} \cdot \text{m}$$

o. From Eq. (15.25),

$$s_m = \pm \frac{0.23}{0.82 + 0.82} = \pm 0.1402$$

From Eq. (15.26),

$$T_{mm} = -T_{mr} = \frac{3 \times 265.58^2}{2 \times 188.5 \times (0.82 + 0.82)} = 342.2 \text{ N} \cdot \text{m}$$

Note: R_s spreads the difference between T_{mm} and T_{mr}. For $R_s = 0$, $T_{mm} = -T_{mr} = 342.2$ N · m, as compared with $T_{mm} = 265.64$ N · m and $T_{mr} = -440.94$ N · m.

15.2.3 Stator Voltage Control

Equation (15.18) indicates that the torque is proportional to the square of the stator supply voltage and a reduction in stator voltage can produce a reduction in speed. If the terminal voltage is reduced to bV_s, Eq. (15.18) gives the developed torque

$$T_d = \frac{3R'_r(bV_s)^2}{s\omega_s[(R_s + R'_r/s)^2 + (X_s + X'_r)^2]}$$

where $b \leq 1$.

Figure 15.4 shows the typical torque–speed characteristics for various values of b. The points of intersection with the load line define the stable operating points. In any magnetic circuit, the induced voltage is proportional to flux and frequency, and the rms air-gap flux can be expressed as

$$V_a = bV_s = K_m\omega\phi$$

or

$$\phi = \frac{V_a}{K_m\omega} = \frac{bV_s}{K_m\omega} \tag{15.31}$$

where K_m is a constant and depends on the number of turns of the stator winding. As the stator voltage is reduced, the air-gap flux and the torque are also reduced. At a

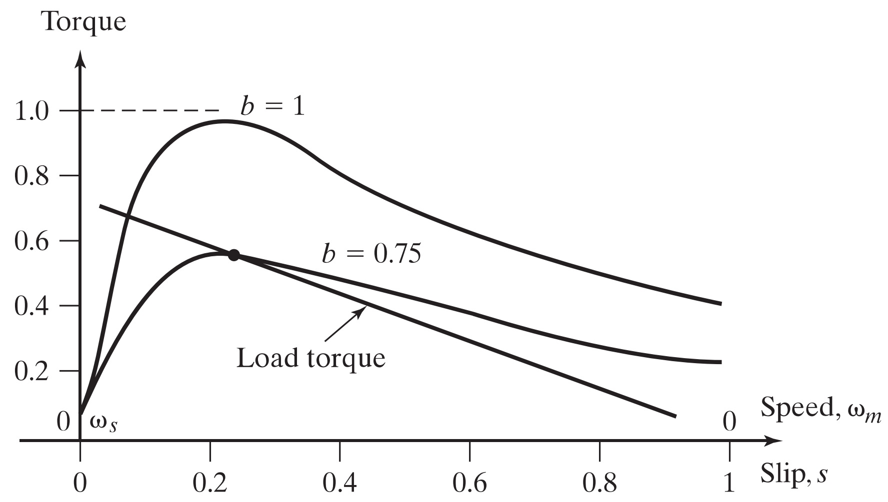


FIGURE 15.4

Torque–speed characteristics with variable stator voltage.

lower voltage, the current can be peaking at a slip of $s_a = \frac{1}{3}$. The range of speed control depends on the slip for maximum torque s_m. For a low-slip motor, the speed range is very narrow. This type of voltage control is not suitable for a constant-torque load and is normally applied to applications requiring low-starting torque and a narrow range of speed at a relatively low slip.

The stator voltage can be varied by three-phase (1) ac voltage controllers, (2) voltage-fed variable dc-link inverters, or (3) pulse-width modulation (PWM) inverters. However, due to limited speed range requirements, the ac voltage controllers are normally used to provide the voltage control. The ac voltage controllers are very simple. However, the harmonic contents are high and the input PF of the controllers is low. They are used mainly in low-power applications, such as fans, blowers, and centrifugal pumps, where the starting torque is low. They are also used for starting high-power induction motors to limit the in-rush current.

The schematic of a reversible phase-controlled induction motor drive is shown in Figure 15.5a. The gating sequence for one direction of rotation is $T_1T_2T_3T_4T_5T_6$ and the gating sequence for reverse rotation is $T_1T_{2r}T_{3r}T_4T_{5r}T_{6r}$. During the reverse direction, the devices T_2, T_3, T_5, and T_6 are not gated. For changing the operation from one direction to the reverse direction, the motor has to be slowed down to zero speed. The triggering angle is delayed so as to produce zero torque, and then the load slows down the rotor. At zero speed, the phase sequence is changed and the triggering angle is retarded until it can produce currents to generate the required torque in the reverse direction. The trajectory for changing the operating point from $P_1\,(\omega_{m1}, T_{e1})$ to $P_2\,(-\omega_{m2}, -T_{e2})$ is shown in Figure 15.5b.

An induction motor can be modeled as an equivalent resistance R_{im} in series with an equivalent reactance X_{im} of an equivalent circuit. Neglecting the effect of R_m in Figure 15.1c, the equivalent parameters can be determined from the motor parameters and the slip.

$$R_{im} = R_s + \frac{X_m^2}{\left(\dfrac{R_r'}{s}\right) + (X_m + X_r')^2}\left(\frac{R_r}{s}\right) \tag{15.32}$$

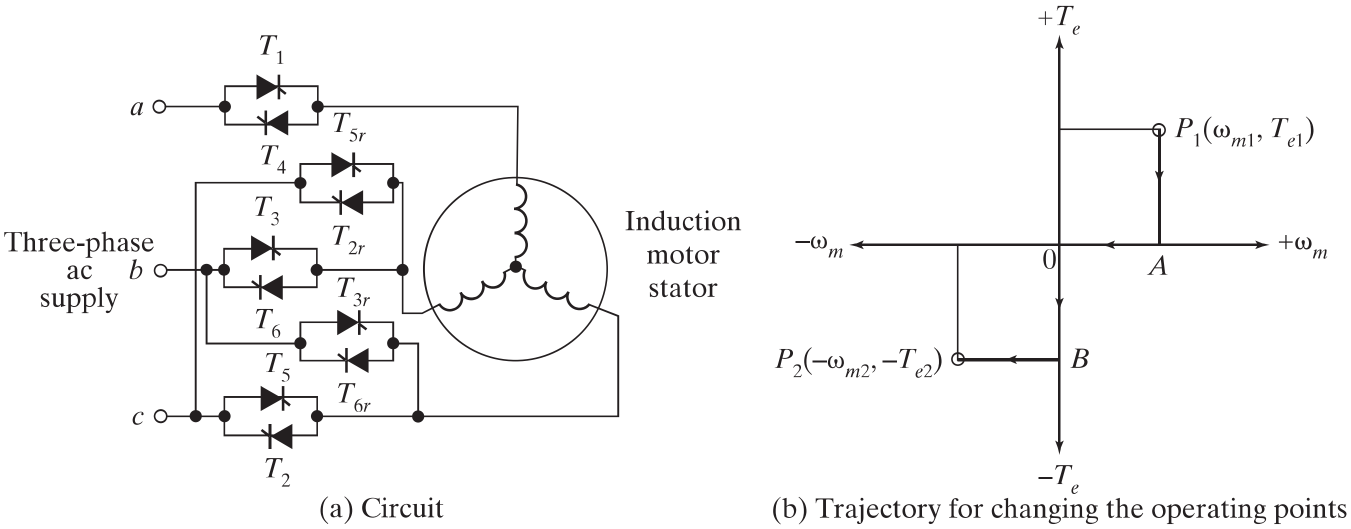


FIGURE 15.5

Reversible phase-controlled induction motor drive.

$$X_{im} = X_s + \frac{\left(\frac{R_r'}{s}\right)^2 + X_r'(X_m + X_r')}{\left(\frac{R_r'}{s}\right)^2 + (X_m + X_r')^2} X_m \tag{15.33}$$

Therefore, the equivalent impedance and power factor angle are given by

$$Z_{im} = \sqrt{R_{im}^2 + X_{im}^2} \tag{15.34}$$

$$\theta = \tan^{-1}\left(\frac{X_{im}}{R_{im}}\right) \tag{15.35}$$

For a triggering delay angle α, the voltage applied to the motor equivalent circuit is given by

$$v(t) = V_m \sin(\omega t + \alpha) \tag{15.36}$$

The corresponding stator current can be expressed as

$$i_{as}(t) = \frac{V_m}{Z_{im}}\left[\sin(\omega_s t + \alpha - \theta) - \sin(\alpha - \theta)e^{\frac{-\omega t}{\tan\theta}}\right] \quad \textit{for} \quad 0 \le \omega t \le \beta \tag{15.37}$$

The conduction angle β is obtained from Eq. (15.37) when the current becomes zero. That is,

$$i_{as}(t) = i_{as}\left(\frac{\beta}{\omega}\right) = 0 \tag{15.38}$$

This gives the nonlinear relationship as

$$\sin(\beta + \alpha - \theta) - \sin(\alpha - \theta)e^{\frac{-\beta}{\tan\theta}} = 0 \tag{15.39}$$

This transcendental equation can be solved for β by an iterative method of solution using Mathcad or Mathlab to find the instantaneous stator current. When the triggering angle α is less than the power factor angle θ, the current will conduct for the positive half-cycle, from α to $(\pi + \alpha)$. During the negative half-cycle starting at $(\pi + \alpha)$ and for $\alpha < \theta$, the positive current is still flowing and the voltage applied to the machine is negative. The stator current in Eq. (15.37) contains harmonic components. As a result, the motor will be subjected to pulsating torques.

Example 15.2 Finding the Performance Parameters of a Three-Phase Induction Motor with Stator Voltage Control

A three-phase, 460-V, 60-Hz, four-pole Y-connected induction motor has the following parameters: $R_s = 1.01\ \Omega$, $R'_r = 0.69\ \Omega$, $X_s = 1.3\ \Omega$, $X'_r = 1.94\ \Omega$, and $X_m = 43.5\ \Omega$. The no-load loss, $P_{\text{no load}}$, is negligible. The load torque, which is proportional to the speed squared, is 41 N · m at 1740 rpm. If the motor speed is 1550 rpm, determine (a) the load torque T_L, (b) the rotor current I_r, (c) the stator supply voltage V_a, (d) the motor input current I_i, (e) the motor input power P_i, (f) the slip for maximum current s_a, (g) the maximum rotor current $I_{r(\max)}$, (h) the speed at maximum rotor current ω_a, and (i) the torque at the maximum current T_a.

Solution

$p = 4$, $f = 60$ Hz, $V_s = 460/\sqrt{3} = 265.58$ V, $R_s = 1.01\ \Omega$, $R'_r = 0.69\ \Omega$, $X_s = 1.3\ \Omega$, $X'_r = 1.94\ \Omega$, $X_m = 43.5\ \Omega$, $\omega = 2\pi \times 60 = 377$ rad/s, and $\omega_s = 377 \times 2/4 = 188.5$ rad/s. Because torque is proportional to speed squared,

$$T_L = K_m \omega_m^2 \tag{15.40}$$

At $\omega_m = 1740\ \pi/30 = 182.2$ rad/s, $T_L = 41$ N · m, and Eq. (15.40) yields $K_m = 41/182.2^2 = 1.235 \times 10^{-3}$ and $\omega_m = 1550\ \pi/30 = 162.3$ rad/s. From Eq. (15.4), $s = (188.5 - 162.3)/188.500 = 0.139$.

a. From Eq. (15.40), $T_L = 1.235 \times 10^{-3} \times 162.3^2 = 32.5$ N · m.

b. From Eqs. (15.10) and (15.12),

$$P_d = 3(I'_r)^2 \frac{R'_r}{s}(1 - s) = T_L \omega_m + P_{\text{no load}} \tag{15.41}$$

For negligible no-load loss,

$$I_r = \left[\frac{s T_L \omega_m}{3R'_r(1 - s)}\right]^{1/2} \tag{15.42}$$

$$= \left[\frac{0.139 \times 32.5 \times 162.3}{3 \times 0.69(1 - 0.139)}\right]^{1/2} = 20.28 \text{ A}$$

c. The stator supply voltage

$$V_a = I'_r\left[\left(R_s + \frac{R'_r}{s}\right)^2 + (X_s + X'_r)^2\right]^{1/2} \tag{15.43}$$

$$= 20.28 \times \left[\left(1.01 + \frac{0.69}{0.139}\right)^2 + (1.3 + 1.94)^2\right]^{1/2} = 137.82$$

d. From Eq. (15.15),

$$\mathbf{Z}_i = \frac{-43.5 \times (1.3 + 1.94) + j43.5 \times (1.01 + 0.69/0.139)}{1.01 + 0.69/0.139 + j(43.5 + 1.3 + 1.94)} = 6.27 \angle 35.82^\circ$$

$$\mathbf{I}_i = \frac{V_a}{\mathbf{Z}_i} = \frac{137.82}{6.27} \angle -144.26^\circ = 22 \angle -35.82^\circ \text{A}$$

e. $\text{PF}_m = \cos(-35.82^\circ) = 0.812$ (lagging). From Eq. (15.13),

$$P_i = 3 \times 137.82 \times 22.0 \times 0.812 = 7386 \text{ W}$$

f. Substituting $\omega_m = \omega_s(1 - s)$ and $T_L = K_m\omega_m^2$ in Eq. (15.42) yields

$$I'_r = \left[\frac{sT_L\omega_m}{3R'_r(1 - s)}\right]^{1/2} = (1 - s)\omega_s\left(\frac{sK_m\omega_s}{3R'_r}\right)^{1/2} \tag{15.44}$$

The slip at which I'_r becomes maximum can be obtained by setting $dI_r/ds = 0$, and this yields

$$s_a = \tfrac{1}{3} \tag{15.45}$$

g. Substituting $s_a = \frac{1}{3}$ in Eq. (15.44) gives the maximum rotor current

$$\begin{aligned} I'_{r(\max)} &= \omega_s\left(\frac{4K_m\omega_s}{81R'_r}\right)^{1/2} \\ &= 188.5 \times \left(\frac{4 \times 1.235 \times 10^{-3} \times 188.5}{81 \times 0.69}\right)^{1/2} = 24.3 \text{ A} \end{aligned} \tag{15.46}$$

h. The speed at the maximum current

$$\begin{aligned} \omega_a &= \omega_s(1-s_a) = (2/3)\omega_s = 0.6667\omega_s \\ &= 188.5 \times 2/3 = 125.27 \text{ rad/s} \quad \text{or} \quad 1200 \text{ rpm} \end{aligned} \tag{15.47}$$

i. From Eqs. (15.9), (15.12a), and (15.44),

$$\begin{aligned} T_a &= 9I_{r(\max)}^2 \frac{R_r}{\omega_s} \\ &= 9 \times 24.3^2 \times \frac{0.69}{188.5} = 19.45 \text{ N}\cdot\text{m} \end{aligned} \tag{15.48}$$

15.2.4 Rotor Voltage Control

In a wound-rotor motor, an external three-phase resistor may be connected to its slip rings, as shown in Figure 15.6a. The developed torque may be varied by varying the resistance R_x. If R_x is referred to the stator winding and added to R_r, Eq. (15.18) may be applied to determine the developed torque. The typical torque–speed characteristics for variations in rotor resistance are shown in Figure 15.6b. This method increases the starting torque while limiting the starting current. However, this is an inefficient

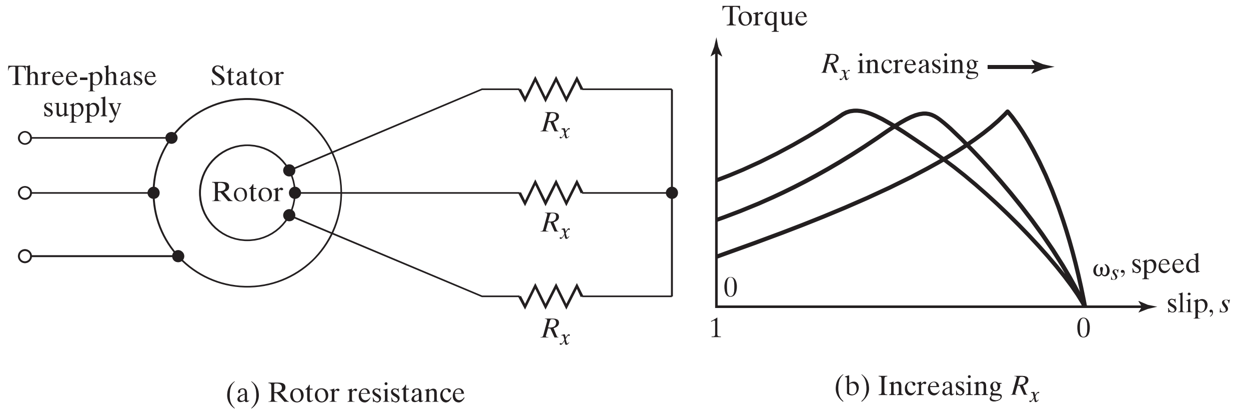


FIGURE 15.6

Speed control by motor resistance.

method and there would be imbalances in voltages and currents if the resistances in the rotor circuit are not equal. A wound-rotor induction motor is designed to have a low-rotor resistance so that the running efficiency is high and the full-load slip is low. The increase in the rotor resistance does not affect the value of maximum torque but increases the slip at maximum torque. The wound-rotor motors are widely used in applications requiring frequent starting and braking with large motor torques (e.g., crane hoists). Because of the availability of rotor windings for changing the rotor resistance, the wound rotor offers greater flexibility for control. However, it increases the cost and needs maintenance due to slip rings and brushes. The wound-rotor motor is less widely used as compared with the squirrel-case motor.

The three-phase resistor may be replaced by a three-phase diode rectifier and a dc converter, as shown in Figure 15.7a, where the gate-turn-off thyristor (GTO) or an insulated-gate bipolar transistor (IGBT) operates as a dc converter switch. The inductor L_d acts as a current source I_d and the dc converter varies the effective resistance, which can be found from Eq. (14.40):

$$R_e = R(1 - k) \tag{15.49}$$

where k is the duty cycle of the dc converter. The speed can be controlled by varying the duty cycle. The portion of the air-gap power, which is not converted into mechanical power, is called *slip power*. The slip power is dissipated in the resistance R.

The slip power in the rotor circuit may be returned to the supply by replacing the dc converter and resistance R, with a three-phase full converter, as shown in Figure 15.7b. The converter is operated in the inversion mode with delay range of $\pi/2 \leq \alpha \leq \pi$, thereby returning energy to the source. The variation of the delay angle permits PF and speed control. This type of drive is known as a *static Kramer* drive. Again, by replacing the bridge rectifiers by three three-phase dual converters (or cycloconverters), as shown in Figure 15.7c, the slip PF in either direction is possible and this arrangement is called a *static Scherbius* drive. The static Kramer and Scherbius drives are used in large power pump and blower applications where limited range of speed control is required. Because the motor is connected directly to the source, the PF of these drives is generally high.

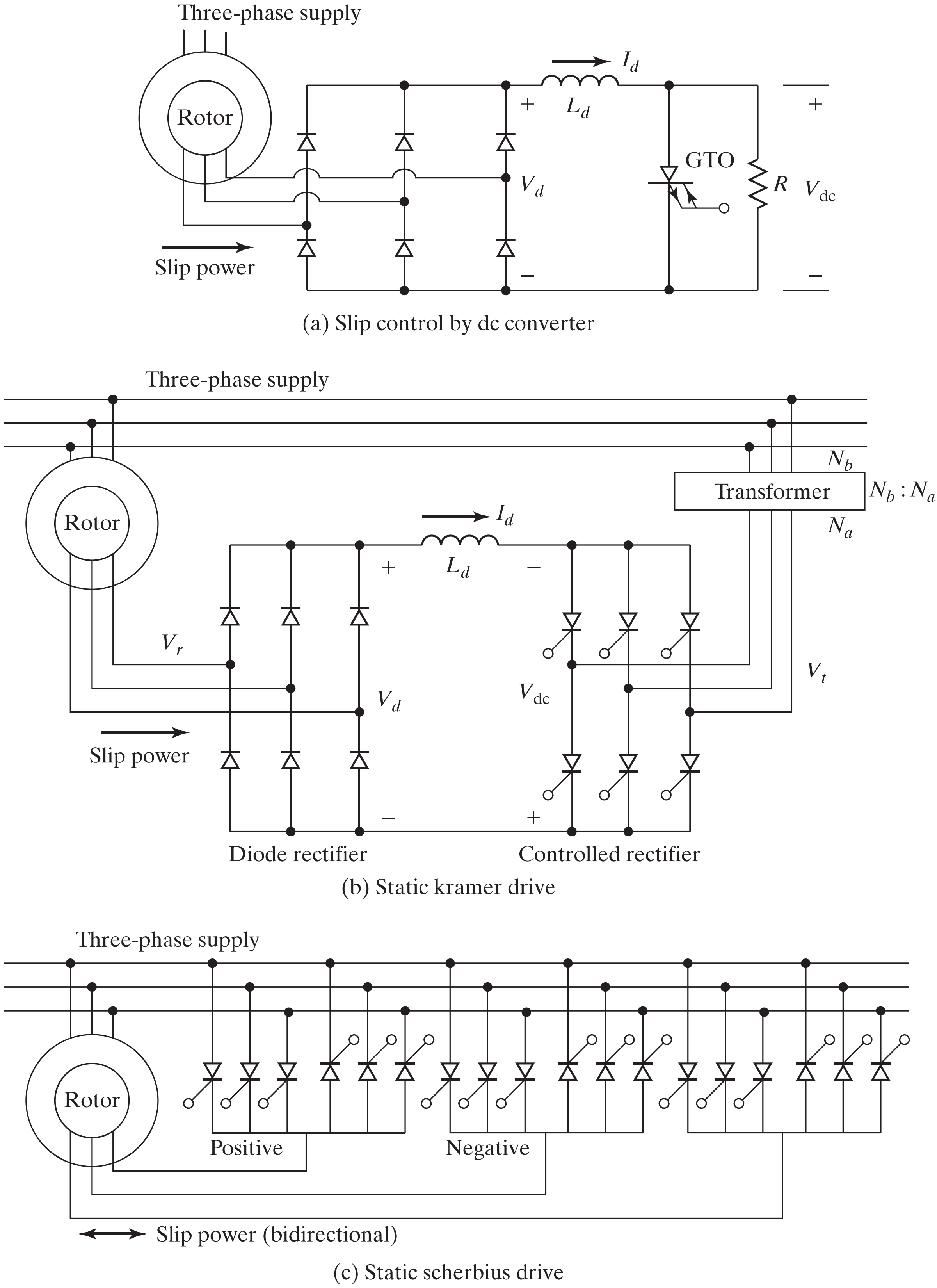


FIGURE 15.7

Slip power control.

Assuming n_r is the effective turns ratio of the stator and the rotor windings, the rotor voltage is related to the stator (and line voltage V_L) by

$$V_r = \frac{sV_L}{n_r} \tag{15.50}$$

The dc output voltage of the three-phase rectifier is

$$V_d = 1.35V_r = \frac{11.35s\,V_L}{n_r} \tag{15.51}$$

Neglecting the resistive voltage in the series inductor L_d,

$$V_d = -V_{\text{dc}} \tag{15.52}$$

V_{dc}, which is the output voltage of a phase-controlled converter, is given by

$$V_{\text{dc}} = 1.35V_t \cos\alpha \tag{15.53}$$

where

$$V_t = \frac{N_a}{N_b}V_L = n_t V_L \tag{15.54}$$

where n_t is the turns ratio of the transformer in the converter side. Using Eq. (15.51) to (15.54), the slip can be found from

$$s = -n_r n_t \cos\alpha \tag{15.55}$$

This gives the delay angle as

$$\alpha = \cos^{-1}\left(\frac{-s}{n_r n_t}\right) \tag{15.56}$$

The delay angle can be varied in the inversion mode from 90° to 180°. But the power switching devices limit the upper range to 155, and thus the practical range of the delay angle is

$$90° \le s \le 155° \tag{15.57}$$

which gives the slip range as

$$0 \le s \le (0.906 \times n_r n_t) \tag{15.58}$$

Example 15.3 Finding the Performance Parameters of a Three-Phase Induction Motor with Rotor Voltage Control

A three-phase, 460-V, 60-Hz, six-pole Y-connected wound-rotor induction motor whose speed is controlled by slip power, as shown in Figure 15.7a, has the following parameters: $R_s = 0.041\ \Omega$, $R_r' = 0.044\ \Omega$, $X_s = 0.29\ \Omega$, $X_r' = 0.44\ \Omega$, and $X_m = 6.1\ \Omega$. The turns ratio of the rotor to stator windings is $n_m = N_r/N_s = 0.9$. The inductance L_d is very large and its current I_d has negligible ripple. The values of R_s, R_r, X_s, and X_r for the equivalent circuit in Figure 15.2 can be considered negligible compared with the effective impedance of L_d. The no-load loss of the motor is negligible. The losses in the rectifier, inductor L_d, and the GTO dc converter are also negligible.

The load torque, which is proportional to speed squared, is 750 N · m at 1175 rpm. (a) If the motor has to operate with a minimum speed of 800 rpm, determine the resistance R. With this value of R, if the desired speed is 1050 rpm, calculate (b) the inductor current I_d, (c) the duty cycle of the dc converter k, (d) the dc voltage V_d, (e) the efficiency, and (f) the input PF_s of the drive.

Solution

$V_a = V_s = 460/\sqrt{3} = 265.58$ V, $p = 6$, $\omega = 2\pi \times 60 = 377$ rad/s, and $\omega_s = 2 \times 377/6 = 125.66$ rad/s. The equivalent circuit of the drive is shown in Figure 15.8a, which is reduced to Figure 15.8b provided the motor parameters are neglected. From Eq. (15.49), the dc voltage at the rectifier output is

$$V_d = I_d R_e = I_d R(1 - k) \tag{15.59}$$

and

$$E_r = sV_s \frac{N_r}{N_s} = sV_s n_m \tag{15.60}$$

For a three-phase rectifier, Eq. (3.33) relates E_r and V_d as

$$V_d = 1.654 \times \sqrt{2}\, E_r = 2.3394 E_r$$

Using Eq. (15.60),

$$V_d = 2.3394 s V_s n_m \tag{15.61}$$

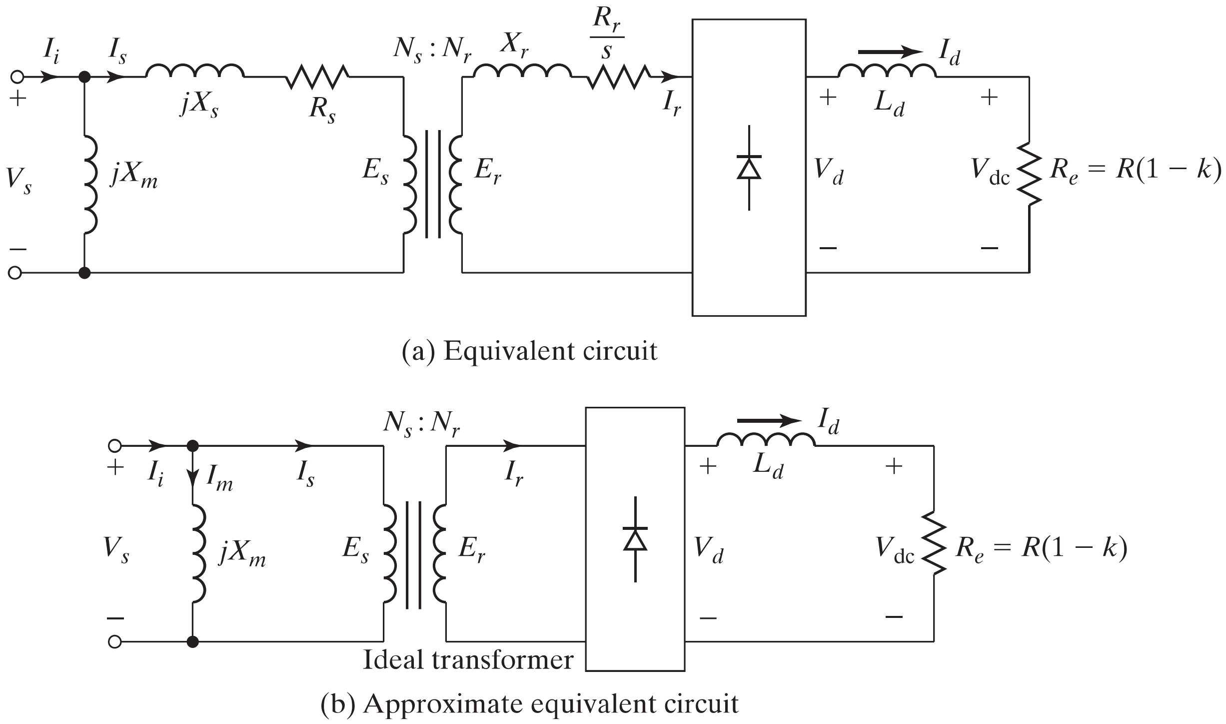


FIGURE 15.8

Equivalent circuits for Example 15.3.

If P_r is the slip power, Eq. (15.9) gives the gap power

$$P_g = \frac{P_r}{s}$$

and Eq. (15.10) gives the developed power as

$$P_d = 3(P_g - P_r) = 3\left(\frac{P_r}{s} - P_r\right) = \frac{3P_r(1 - s)}{s} \tag{15.62}$$

Because the total slip power is $3P_r = V_d I_d$ and $P_d = T_L \omega_m$, Eq. (15.62) becomes

$$P_d = \frac{(1 - s)V_d I_d}{s} = T_L \omega_m = T_L \omega_s (1 - s) \tag{15.63}$$

Substituting V_d from Eq. (15.61) in Eq. (15.63) and solving for I_d gives

$$I_d = \frac{T_L \omega_s}{2.3394 V_s n_m} \tag{15.64}$$

which indicates that the inductor current is independent of the speed. Equating Eq. (15.59) to Eq. (15.61) gives

$$2.3394 s V_s n_m = I_d R(1 - k)$$

which gives

$$s = \frac{I_d R(1 - k)}{2.3394\ V_s n_m} \tag{15.65}$$

The speed can be found from Eq. (15.65) as

$$\omega_m = \omega_s(1 - s) = \omega_s\left[1 - \frac{I_d R(1 - k)}{2.3394 V_s n_m}\right] \tag{15.66}$$

$$= \omega_s\left[1 - \frac{T_L \omega_s R(1 - k)}{(2.3394 V_s n_m)^2}\right] \tag{15.67}$$

which shows that for a fixed duty cycle, the speed decreases with load torque. By varying k from 0 to 1, the speed can be varied from a minimum value to ω_s.

a. $\omega_m = 800\ \pi/30 = 83.77$ rad/s. From Eq. (15.40) the torque at 900 rpm is

$$T_L = 750 \times \left(\frac{800}{1175}\right)^2 = 347.67\ \text{N}\cdot\text{m}$$

From Eq. (15.64), the corresponding inductor current is

$$I_d = \frac{347.67 \times 125.66}{2.3394 \times 265.58 \times 0.9} = 78.13\ \text{A}$$

The speed is minimum when the duty cycle k is zero and Eq. (15.66) gives the minimum speed,

$$83.77 = 125.66\left(1 - \frac{78.13R}{2.3394 \times 265.58 \times 0.9}\right)$$

and this yields $R = 2.3856\ \Omega$.

b. At 1050 rpm

$$T_L = 750 \times \left(\frac{1050}{1175}\right)^2 = 598.91 \text{ N}\cdot\text{m}$$

$$I_d = \frac{598.91 \times 125.66}{2.3394 \times 265.58 \times 0.9} = 134.6 \text{ A}$$

c. $\omega_m = 1050\,\pi/30 = 109.96$ rad/s and Eq. (15.66) gives

$$109.96 = 125.66\left[1 - \frac{134.6 \times 2.3856(1-k)}{2.3394 \times 265.58 \times 0.9}\right]$$

which gives $k = 0.782$.

d. Using Eq. (15.4), the slip is

$$s = \frac{125.66 - 109.96}{125.66} = 0.125$$

From Eq. (15.61),

$$V_d = 2.3394 \times 0.125 \times 265.58 \times 0.9 = 69.9 \text{ V}$$

e. The power loss,

$$P_1 = V_d I_d = 69.9 \times 134.6 = 9409 \text{ W}$$

The output power,

$$P_o = T_L \omega_m = 598.91 \times 109.96 = 65{,}856 \text{ W}$$

The rms rotor current referred to the stator is

$$I'_r = \sqrt{\frac{2}{3}}\, I_d n_m = \sqrt{\frac{2}{3}} \times 134.6 \times 0.9 = 98.9 \text{ A}$$

The rotor copper loss is $P_{ru} = 3 \times 0.044 \times 98.9^2 = 1291$ W, and the stator copper loss is $P_{su} = 3 \times 0.041 \times 98.9^2 = 1203$ W. The input power is

$$P_i = 65{,}856 + 9409 + 1291 + 1203 = 77{,}759 \text{ W}$$

The efficiency is 65,856/77,759 = 85%.

f. From Eq. (10.19) for $n = 1$, the fundamental component of the rotor current referred to the stator is

$$I'_{r1} = 0.7797 I_d \frac{N_r}{N_s} = 0.7797 I_d n_m$$

$$= 0.7797 \times 134.6 \times 0.9 = 94.45 \text{ A}$$

and the rms current through the magnetizing branch is

$$I_m = \frac{V_a}{X_m} = \frac{265.58}{6.1} = 43.54 \text{ A}$$

The rms fundamental component of the input current is

$$I_{i1} = \left[(0.7797 I_d n_m)^2 + \left(\frac{V_a}{X_m}\right)^2 \right]^{1/2} \tag{15.68}$$

$$= (94.45^2 + 43.54^2)^{1/2} = 104 \text{ A}$$

The PF angle is given approximately by

$$\theta_m = -\tan^{-1} \frac{V_a/X_m}{0.7797 I_d n_m} \tag{15.69}$$

$$= -\tan^{-1} \frac{43.54}{94.45} = \underline{/-24.74°}$$

The input PF is $PF_s = \cos(-24.74°) = 0.908$ (lagging).

Example 15.4 Finding the Performance Parameters of a Static Kramer Drive

The induction motor in Example 15.3 is controlled by a static Kramer drive, as shown in Figure 15.7b. The turns ratio of the converter ac voltage to supply voltage is $n_c = N_a/N_b = 0.40$. The load torque is 750 N · m at 1175 rpm. If the motor is required to operate at a speed of 1050 rpm, calculate (a) the inductor current I_d; (b) the dc voltage V_d; (c) the delay angle of the converter α; (d) the efficiency; and (e) the input PF of the drive, PF_s. The losses in the diode rectifier, converter, transformer, and inductor L_d are negligible.

Solution

$V_a = V_s = 460/\sqrt{3} = 265.58$ V, $p = 6$, $\omega = 2\pi \times 60 = 377$ rad/s, $\omega_s = 2 \times 377/6 = 125.66$ rad/s, and $\omega_m = 1050\ \pi/30 = 109.96$ rad/s. Then

$$s = \frac{125.66 - 109.96}{125.66} = 0.125$$

$$T_L = 750 \times \left(\frac{1050}{1175}\right)^2 = 598.91 \text{ N} \cdot \text{m}$$

a. The equivalent circuit of the drive is shown in Figure 15.9, where the motor parameters are neglected. From Eq. (15.64), the inductor current is

$$I_d = \frac{598.91 \times 125.66}{2.3394 \times 265.58 \times 0.9} = 134.6 \text{ A}$$

b. From Eq. (15.61),

$$V_d = 2.3394 \times 0.125 \times 265.58 \times 0.9 = 69.9 \text{ V}$$

c. Because the ac input voltage to the converter is $V_c = n_c V_s$, Eq. (10.15) gives the average voltage at the dc side of the converter as

$$V_{dc} = -\frac{3\sqrt{3}\sqrt{2}\, n_c V_s}{\pi} \cos\alpha = -2.3394 n_c V_s \cos\alpha \tag{15.70}$$

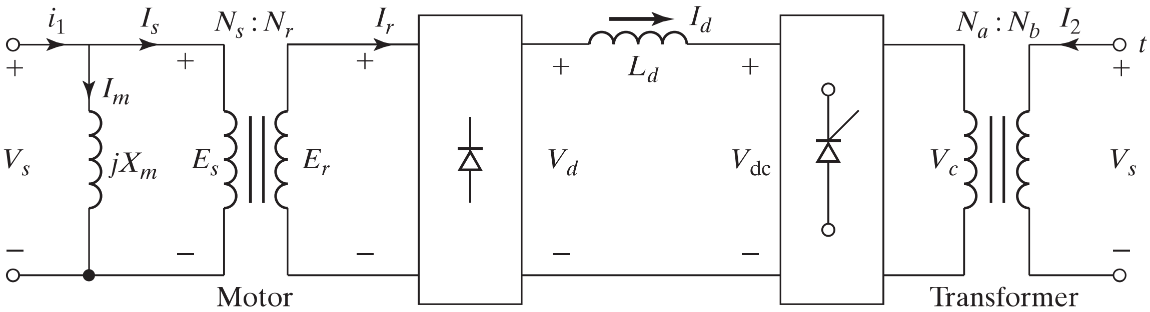


FIGURE 15.9

Equivalent circuit for static Kramer drive.

Because $V_d = V_{dc}$, Eqs. (15.61) and (15.70) give

$$2.3394 s V_s n_m = -2.3394 n_c V_s \cos \alpha$$

which gives

$$s = \frac{-n_c \cos \alpha}{n_m} \tag{15.71}$$

The speed, which is independent of torque, becomes

$$\omega_m = \omega_s(1 - s) = \omega_s\left(1 + \frac{n_c \cos \alpha}{n_m}\right) \tag{15.72}$$

$$109.96 = 125.66 \times \left(1 + \frac{0.4 \cos \alpha}{0.9}\right)$$

which gives the delay angle, $\alpha = 106.3°$.

d. The power fed back

$$P_1 = V_d I_d = 69.9 \times 134.6 = 9409 \text{ W}$$

The output power

$$P_o = T_L \omega_m = 598.91 \times 109.96 = 65{,}856 \text{ W}$$

The rms rotor current referred to the stator is

$$I'_r = \sqrt{\frac{2}{3}}\, I_d n_m = \sqrt{\frac{2}{3}} \times 134.6 \times 0.9 = 98.9 \text{ A}$$

$$P_{ru} = 3 \times 0.044 \times 98.9^2 = 1291 \text{ W}$$

$$P_{su} = 3 \times 0.041 \times 98.9^2 = 1203 \text{ W}$$

$$P_i = 65{,}856 + 1291 + 1203 = 68{,}350 \text{ W}$$

The efficiency is 65,856/68,350 = 96%.

e. From (f) in Example 15.3, $I'_{r1} = 0.7797 I_d n_m = 94.45$ A, $I_m = 265.58/6.1 = 43.54$ A, and $\mathbf{I}_{i1} = 104\underline{/-24.74°}$. From Example 10.5, the rms current fed back to the supply is

$$\mathbf{I}_{i2} = \sqrt{\frac{2}{3}}\, I_d n_c \underline{/-\alpha} = \sqrt{\frac{2}{3}} \times 134.6 \times 0.4 \underline{/-\alpha} = 41.98 \underline{/-106.3°}$$

The effective input current of the drive is

$$\mathbf{I}_i = \mathbf{I}_{i1} + \mathbf{I}_{i2} = 104\,\angle{-24.74^\circ} + 41.98\,\angle{-106.3^\circ} = 117.7\,\angle{-45.4^\circ}\text{ A}$$

The input PF is $PF_s = \cos(-45.4^\circ) = 0.702$ (lagging).

Note: The efficiency of this drive is higher than that of the rotor resistor control by a dc converter. The PF is dependent on the turns ratio of the transformer (e.g., if $n_c = 0.9$, $\alpha = 97.1^\circ$ and $PF_s = 0.5$; if $n_c = 0.2$, $\alpha = 124.2^\circ$ and $PF_s = 0.8$).

15.2.5 Frequency Control

The torque and speed of induction motors can be controlled by changing the supply frequency. We can notice from Eq. (15.31) that at the rated voltage and rated frequency, the flux is the rated value. If the voltage is maintained fixed at its rated value while the frequency is reduced below its rated value, the flux increases. This would cause saturation of the air-gap flux, and the motor parameters would not be valid in determining the torque–speed characteristics. At low frequency, the reactances decrease and the motor current may be too high. This type of frequency control is not normally used.

If the frequency is increased above its rated value, the flux and torque would decrease. If the synchronous speed corresponding to the rated frequency is called the *base speed* ω_b, the synchronous speed at any other frequency becomes

$$\omega_s = \beta\omega_b$$

and

$$s = \frac{\beta\omega_b - \omega_m}{\beta\omega_b} = 1 - \frac{\omega_m}{\beta\omega_b} \tag{15.73}$$

The torque expression in Eq. (15.18) becomes

$$T_d = \frac{3R'_r V_a^2}{s\beta\omega_b[(R_s + R'_r/s)^2 + (\beta X_s + \beta X'_r)^2]} \tag{15.74}$$

The typical torque–speed characteristics are shown in Figure 15.10 for various values of β. The three-phase inverter in Figure 6.6a can vary the frequency at a fixed voltage. If R_s is negligible, Eq. (15.26) gives the maximum torque at the base speed as

$$T_{mb} = \frac{3V_a^2}{2\omega_b(X_s + X'_r)} \tag{15.75}$$

The maximum torque at any other frequency is

$$T_m = \frac{3}{2\omega_b(X_s + X'_r)}\left(\frac{V_a}{\beta}\right)^2 \tag{15.76}$$

and from Eq. (15.25), the corresponding slip is

$$s_m = \frac{R'_r}{\beta(X_s + X'_r)} \tag{15.77}$$

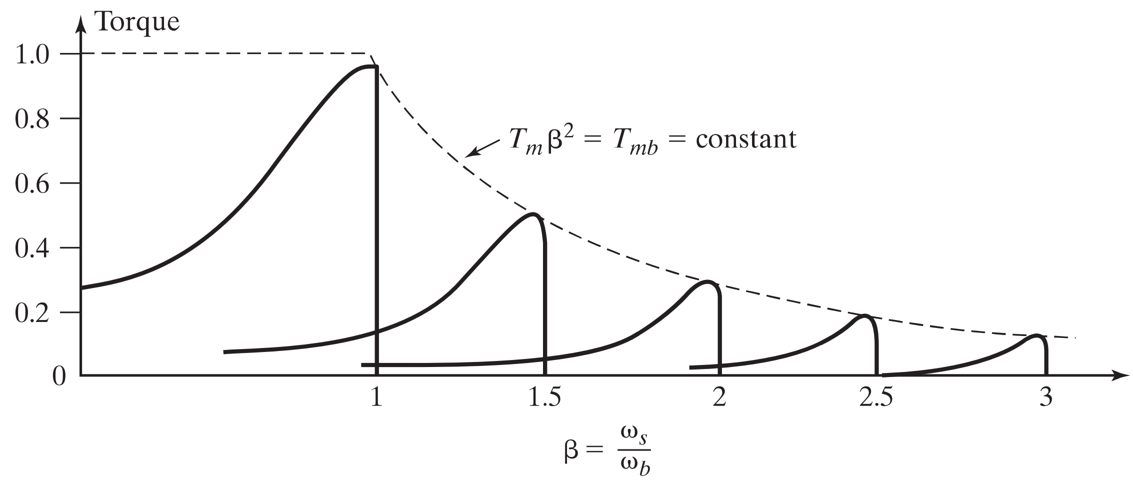


FIGURE 15.10

Torque characteristics with frequency control.

Normalizing Eq. (15.76) with respect to Eq. (15.75) yields

$$\frac{T_m}{T_{mb}} = \frac{1}{\beta^2} \tag{15.78}$$

and

$$T_m\beta^2 = T_{mb} \tag{15.79}$$

Thus, from Eqs. (15.78) and (15.79), it can be concluded that the maximum torque is inversely proportional to frequency squared, and $T_m\beta^2$ remains constant, similar to the behavior of dc series motors. In this type of control, the motor is said to be operated in a *field-weakening mode*. For $\beta > 1$, the motor is operated at a constant terminal voltage and the flux is reduced, thereby limiting the torque capability of the motor. For $1 < \beta < 1.5$, the relation between T_m and β can be considered approximately linear. For $\beta < 1$, the motor is normally operated at a constant flux by reducing the terminal voltage V_a along with the frequency so that the flux remains constant.

Example 15.5 Finding the Performance Parameters of a Three-Phase Induction Motor with Frequency Control

A three-phase, 11.2-kW, 1750-rpm, 460-V, 60-Hz, four-pole Y-connected induction motor has the following parameters: $R_s = 0$, $R'_r = 0.38\ \Omega$, $X_s = 1.14\ \Omega$, $X'_r = 1.71\ \Omega$, and $X_m = 33.2\ \Omega$. The motor is controlled by varying the supply frequency. If the breakdown torque requirement is 35 N · m, calculate (a) the supply frequency and (b) the speed ω_m at the maximum torque.

Solution

$V_a = V_s = 460/\sqrt{3} = 258 \times 58$ V, $\omega_b = 2\pi \times 60 = 377$ rad/s, $p = 4$, $P_0 = 11{,}200$ W, $T_{mb} \times 1750\ \pi/30 = 11{,}200$, $T_{mb} = 61.11$ N · m, and $T_m = 35$ N · m.

a. From Eq. (15.79),

$$\beta = \sqrt{\frac{T_{mb}}{T_m}} = \sqrt{\frac{61.11}{35}} = 1.321$$

$$\omega_s = \beta\omega_b = 1.321 \times 377 = 498.01 \text{ rad/s}$$

From Eq. (15.1), the supply frequency is

$$\omega = \frac{4 \times 498.01}{2} = 996 \text{ rad/s} \quad \text{or} \quad 158.51 \text{ Hz}$$

b. From Eq. (15.77), the slip for maximum torque is

$$s_m = \frac{R'_r/\beta}{X_s + X'_r} = \frac{0.38/1.321}{1.14 + 1.71} = 0.101$$

$$\omega_m = 498.01 \times (1 - 0.101) = 447.711 \text{ rad/s} \quad \text{or} \quad 4275 \text{ rpm}$$

Note: This solution uses the rated power and the speed to calculate T_{mb}. Alternatively, we could substitute the rated voltage and the motor parameters in Eqs. (15.75), (15.78) and (15.77) to find T_{mb}, β and s_m, We may get different results because the motor ratings and parameters are not accurately related and dimensioned. The parameters were selected arbitrarily in this example. Both approaches would be correct.

15.2.6 Voltage and Frequency Control

If the ratio of voltage to frequency is kept constant, the flux in Eq. (15.31) remains constant. Equation (15.76) indicates that the maximum torque, which is independent of frequency, can be maintained approximately constant. However, at a high frequency, the air-gap flux is reduced due to the drop in the stator impedance and the voltage has to be increased to maintain the torque level. This type of control is usually known as *volts/hertz* control.

If $\omega_s = \beta\omega_b$, and the voltage-to-frequency ratio is constant so that

$$\frac{V_a}{\omega_s} = d \tag{15.80}$$

The ratio d, which is determined from the rated terminal voltage V_s and the base speed ω_b, is given by

$$d = \frac{V_s}{\omega_b} \tag{15.81}$$

From Eqs. (15.80) and (15.81), we get

$$V_a = d\omega_s = \frac{V_s}{\omega_b}\beta\,\omega_b = V_s\omega_b \tag{15.82}$$

Substituting V_a from Eq. (15.80) into Eq. (15.74) yields the torque T_d, and the slip for maximum torque is

$$s_m = \frac{R'_r}{[R_s^2 + \beta^2(X_s + X'_r)^2]^{1/2}} \tag{15.83}$$

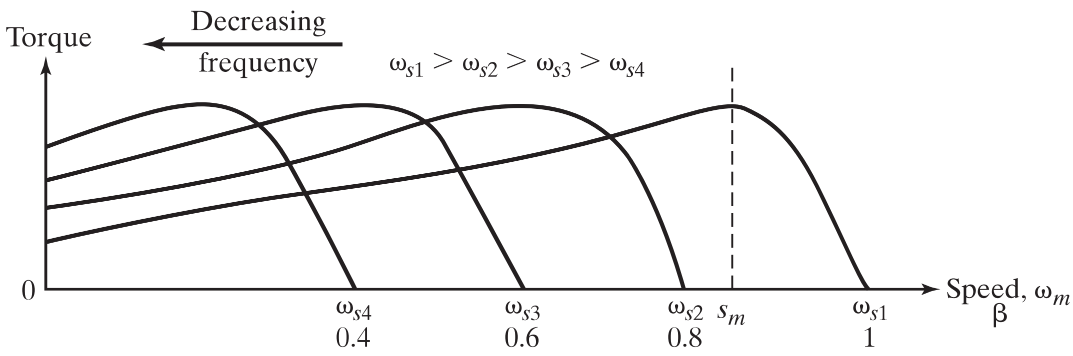


FIGURE 15.11

Torque–speed characteristics with volts/hertz control.

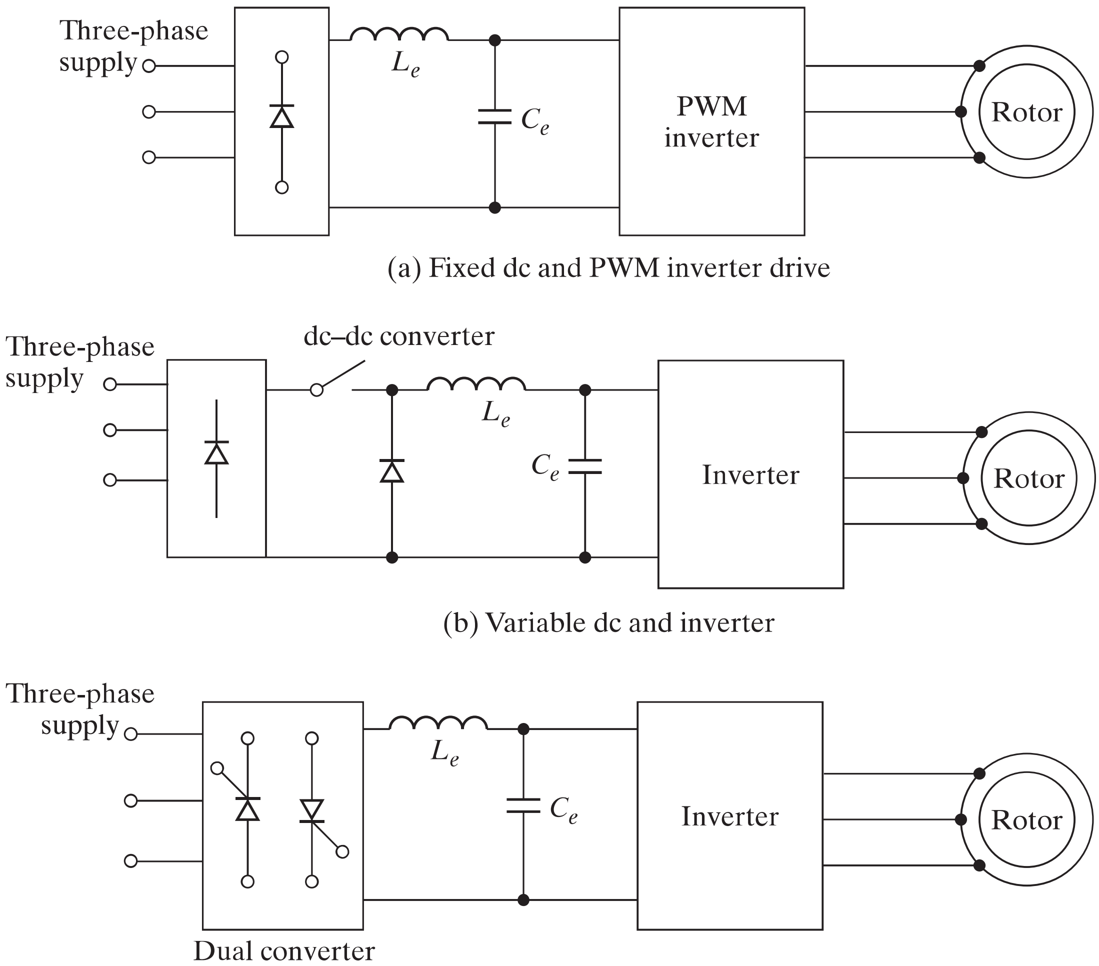
The typical torque–speed characteristics are shown in Figure 15.11. As the frequency is reduced, β decreases and the slip for maximum torque increases. For a given torque demand, the speed can be controlled according to Eq. (15.81) by changing the frequency. Therefore, by varying both the voltage and frequency, the torque and speed can be controlled. The torque is normally maintained constant while the speed is varied. The voltage at variable frequency can be obtained from three-phase inverters or cycloconverters. The cycloconverters are used in very large power applications (e.g., locomotives and cement mills), where the frequency requirement is one-half or one-third of the line frequency.

Three possible circuit arrangements for obtaining variable voltage and frequency are shown in Figure 15.12. In Figure 15.12a, the dc voltage remains constant and the PWM techniques are applied to vary both the voltage and frequency within the inverter. Due to diode rectifier, regeneration is not possible and the inverter would generate harmonics into the ac supply. In Figure 15.12b, the dc–dc converter varies the dc voltage to the inverter and the inverter controls the frequency. Due to the dc converter, the harmonic injection into the ac supply is reduced. In Figure 15.12c, the dc voltage is varied by the dual converter and frequency is controlled within the inverter. This arrangement permits regeneration; however, the input PF of the converter is low, especially at a high delay angle.

The control implementation of the volts/hertz strategy for the circuit arrangement in Figure 15.12a is shown in Figure 15.13 [23]. The electrical rotor speed ω_r is compared with its commanded value ω_r^* and the error is processed through a controller, usually a P_I, and a limiter to obtain the slip-speed command, ω_{s1}^*. ω_r is related to the mechanical motor speed by $\omega_r = (p/2)\,\omega_m$. The limiter ensures that the slip-speed command is within the maximum allowable slip speed of the induction motor. The slip-speed command is added to electrical rotor speed to obtain the stator frequency command, $\omega = \omega_{sl} + \omega_r$. Thereafter, the stator frequency command f is processed as in an open-loop drive.

Using the equivalent circuit in Figure 15.1b, we get the per-phase stator voltage

$$V_a = V_s = I_s(R_s + jX_s) + V_m = I_s(R_s + jX_s) + j(\lambda_m I_M)\omega \tag{15.84}$$
$$= I_s R_s + j(I_s X_s + \lambda_m I_M \omega) = I_s R_s + j\omega_s(I_s L_s + \lambda_m I_M)$$



(a) Fixed dc and PWM inverter drive

(b) Variable dc and inverter

(c) Variable dc from dual converter and inverter

FIGURE 15.12

Voltage-source induction motor drives.

This can be normalized to per-unit value as given by

$$V_{an} = I_{sn}R_{sn} + j\omega_n(I_{sn}L_{sn} + \lambda_{mn}) \tag{15.84a}$$

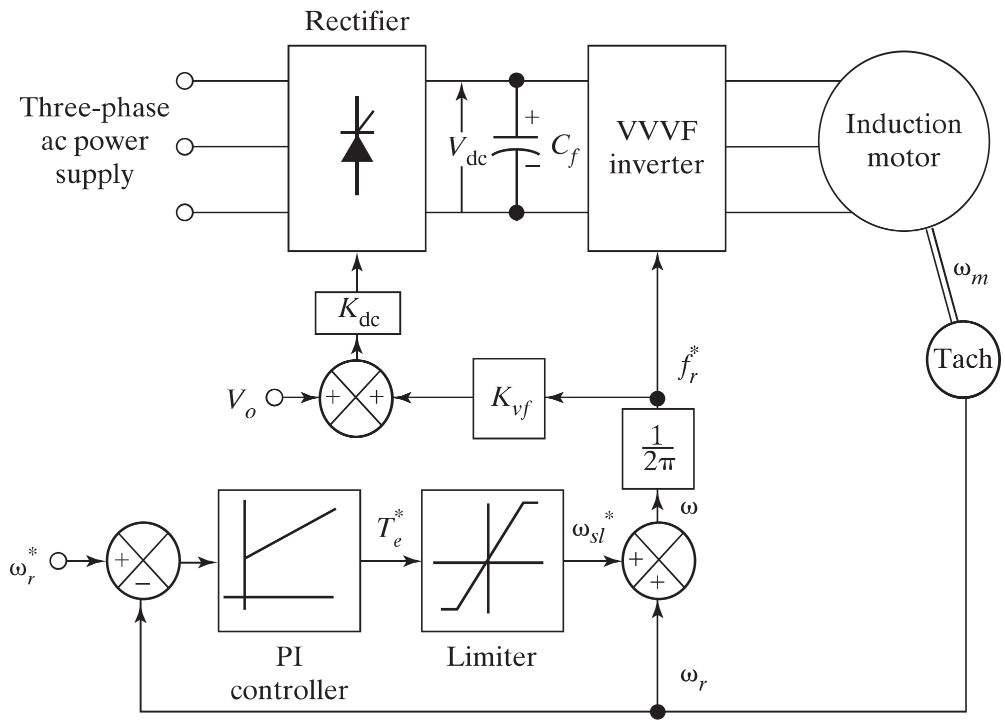
where $\omega_n = \dfrac{\omega}{\omega_b}$; $I_{sn} = \dfrac{I_s}{I_b}$; $R_{sn} = \dfrac{I_b R_s}{V_b}$; $L_{sn} = \dfrac{I_b L_s \omega_s}{V_b \omega_b}$; $\lambda_{mn} = \dfrac{\lambda_m}{\lambda_b}$

Therefore, the magnitude of the normalized input-phase stator voltage is given by

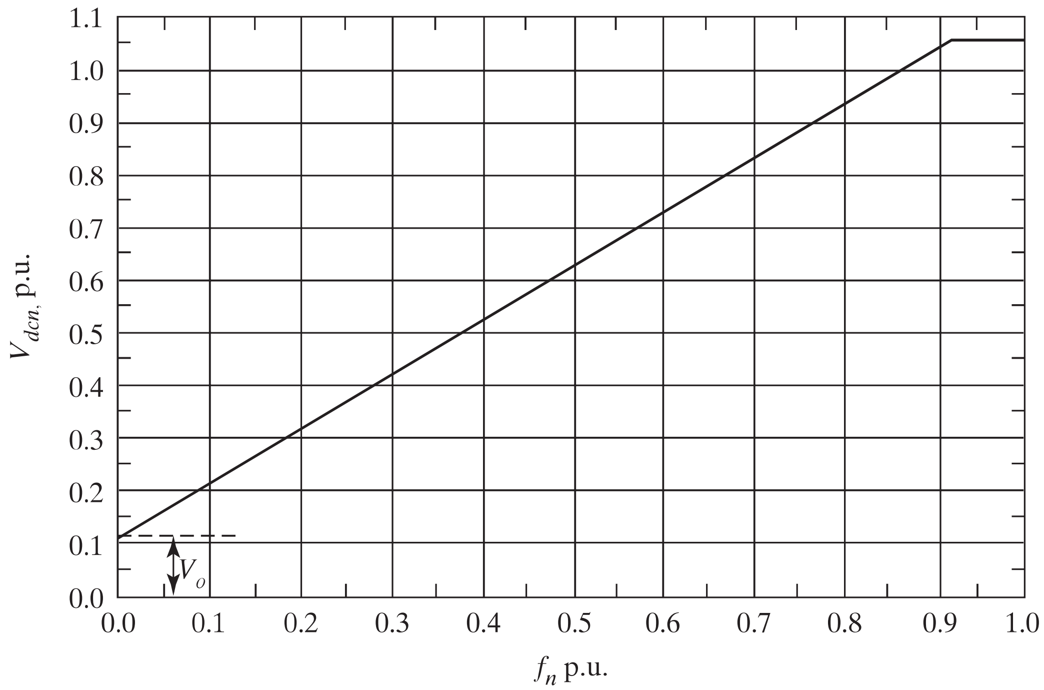
$$V_{an} = \sqrt{(I_{sn}R_{sn})^2 + \omega_n^2(I_{sn}L_{sn} + \lambda_{mn})^2} \tag{15.85}$$

The input voltage is dependent on the frequency, the air-gap flux magnitude, the stator impedance, and the magnitude of the stator current. It can be shown by plotting this relationship that is approximately linear and can be approximated by a preprogrammed volt-to-frequency relationship as given by

$$V_a = I_s R_s + K_{vf} f = V_o + K_{vf} f \tag{15.86}$$



(a) Block diagram



(b) Programmed voltage-to-frequency profile

FIGURE 15.13

Block diagram of V–V inverter for implementation of volts/hertz control strategy [23].

K_{vf} is the volt-to-frequency constant for a given flux and can be found from Eq. (15.31) as given by

$$K_{vf} = \frac{V_a}{f} = \frac{1}{2\pi K_m \phi}\left(\frac{V_a}{f}\right) \tag{15.87}$$

The stator voltage V_a in Eq. (15.86) equals the phase voltage V_{ph} of the three-phase inverter and is related to the dc-link voltage V_{dc} by

$$V_a = V_{ph} = \frac{2}{\pi}\frac{V_{dc}}{\sqrt{2}} = 0.45V_{dc} \tag{15.88}$$

Equating Eq. (15.86) to Eq. (15.88), we get

$$0.45V_{dc} = V_o + V_m(=E_g) = V_o + K_{vf}f \tag{15.89}$$

This can be expressed in a normalized form as

$$0.45V_{dcn} = V_{on} + E_{gn} = V_{on} + f_n \tag{15.90}$$

where $$V_{dcn} = \frac{V_{dc}}{V_a};\ V_{on} = \frac{V_o}{V_a};\ E_{gn} = \frac{K_{vf}f}{K_{vf}f_b} = f_s;\ f_n = \frac{f}{f_b}$$

$K_{dc} = 0.45$ is the constant of proportionality between the dc load voltage and the stator frequency. A typical normalized relationship is shown in Figure 15.13b.

Example 15.6 Finding the Performance Parameters of a Three-Phase Induction Motor with Voltage and Frequency Control

A three-phase, 11.2-kW, 1750-rpm, 460-V, 60-Hz, four-pole, Y-connected induction motor has the following parameters: $R_s = 0.66\ \Omega$, $R'_r = 0.38\ \Omega$, $X_s = 1.14\ \Omega$, $X'_r = 1.71\ \Omega$, and $X_m = 33.2\ \Omega$. The motor is controlled by varying both the voltage and frequency. The volts/hertz ratio, which corresponds to the rated voltage and rated frequency, is maintained constant. (a) Calculate the maximum torque T_m and the corresponding speed ω_m for 60 and 30 Hz. (b) Repeat (a) if R_s is negligible.

Solution

$p = 4$, $V_a = V_s = 460/\sqrt{3} = 265.58$ V, $\omega = 2\pi \times 60 = 377$ rad/s, and from Eq. (15.1), $\omega_b = 2 \times 377/4 = 188.5$ rad/s. From Eq. (15.80), $d = 265.58/188.5 = 1.409$.

a. At 60 Hz, $\omega_b = \omega_s = 188.5$ rad/s, $\beta = 1$, and $V_a = d\omega_s = 1.409 \times 188.5 = 265.58$ V. From Eq. (15.83),

$$s_m = \frac{0.38}{[0.66^2 + (1.14 + 1.71)^2]^{1/2}} = 0.1299$$

$$\omega_m = 188.5 \times (1 - 0.1299) = 164.01 \text{ rad/s} \quad \text{or} \quad 1566 \text{ rpm}$$

From Eq. (15.21), the maximum torque is

$$T_m = \frac{3 \times 265.58^2}{2 \times 188.5 \times [0.66 + \sqrt{0.66^2 + (1.14 + 1.71)^2}]} = 156.55 \text{ N}\cdot\text{m}$$

At 30 Hz, $\omega_s = 2 \times 2 \times \pi\, 30/4 = 94.25$ rad/s, $\beta = 30/60 = 0.5$, and $V_a = d\omega_s = 1.409 \times 94.25 = 132.79$ V. From Eq. (15.83), the slip for maximum torque is

$$s_m = \frac{0.38}{[0.66^2 + 0.5^2 \times (1.14 + 1.71)^2]^{1/2}} = 0.242$$

$$\omega_m = 94.25 \times (1 - 0.242) = 71.44 \text{ rad/s} \quad \text{or} \quad 682 \text{ rpm}$$

$$T_m = \frac{3 \times 132.79^2}{2 \times 94.25 \times [0.66 + \sqrt{0.66^2 + 0.5^2 \times (1.14 + 1.71)^2}]} = 125.82 \text{ N} \cdot \text{m}$$

b. At 60 Hz, $\omega_b = \omega_s = 188.5$ rad/s and $V_a = 265.58$ V. From Eq. (15.77),

$$s_m = \frac{0.38}{1.14 + 1.71} = 0.1333$$

$$\omega_m = 188.5 \times (1 - 0.1333) = 163.36 \text{ rad/s} \quad \text{or} \quad 1560 \text{ rpm}$$

From Eq. (15.76), the maximum torque is $T_m = 196.94$ N · m.

At 30 Hz, $\omega_s = 94.25$ rad/s, $\beta = 0.5$, and $V_a = 132.79$ V. From Eq. (15.77),

$$s_m = \frac{0.38/0.5}{1.14 + 1.71} = 0.2666$$

$$\omega_m = 94.25 \times (1 - 0.2666) = 69.11 \text{ rad/s} \quad \text{or} \quad 660 \text{ rpm}$$

From Eq. (15.76), the maximum torque is $T_m = 196.94$ N · m.

Note: Neglecting R_s may introduce a significant error in the torque estimation, especially at a low frequency.

15.2.7 Current Control

The torque of induction motors can be controlled by varying the rotor current. The input current, which is readily accessible, is varied instead of the rotor current. For a fixed input current, the rotor current depends on the relative values of the magnetizing and rotor circuit impedances. From Figure 15.2, the rotor current can be found as

$$\bar{\mathbf{I}}'_r = \frac{jX_m I_i}{R_s + R'_r/s + j(X_m + X_s + X'_r)} = I'_r \angle \theta_1 \tag{15.91}$$

From Eqs. (15.9) and (15.12a), the developed torque is

$$T_d = \frac{3R'_r(X_m I_i)^2}{s\omega_s[(R_s + R'_r/s)^2 + (X_m + X_s + X'_r)^2]} \tag{15.92}$$

and the starting torque at $s = 1$ is

$$T_s = \frac{3R'_r(X_m I_i)^2}{\omega_s[(R_s + R'_r)^2 + (X_m + X_s + X'_r)^2]} \tag{15.93}$$

The slip for maximum torque is

$$s_m = \pm \frac{R'_r}{[R_s^2 + (X_m + X_s + X'_r)^2]^{1/2}} \tag{15.94}$$

In a real situation, as shown in Figure 15.1b and 15.1c, the stator current through R_s and X_s is constant at I_i. Generally, X_m is much greater than X_s and R_s, which can be neglected for most applications. Neglecting the values of R_s and X_s, Eq. (15.94) becomes

$$s_m = \pm \frac{R'_r}{X_m + X'_r} \tag{15.95}$$

and at $s = s_m$, Eq. (15.92) gives the maximum torque,

$$T_m = \frac{3X_m^2}{2\omega_s(X_m + X'_r)} I_i^2 = \frac{3L_m^2}{2(L_m + L'_r)} I_i^2 \tag{15.96}$$

The input current I_i is supplied from a dc current source I_d consisting of a large inductor. The fundamental stator rms phase current of the three-phase current-source inverter is related to I_d by

$$I_i = I_s = \frac{\sqrt{2}\sqrt{3}}{\pi} I_d \tag{15.97}$$

It can be noticed from Eq. (15.96) that the maximum torque depends on the square of the current and is approximately independent of the frequency. The typical torque–speed characteristics are shown in Figure 15.14a for increasing values of stator current. Because X_m is large as compared with X_s and X'_r, the starting torque is low. As the speed increases (or slip decreases), the stator voltage rises and the torque increases. The starting current is low due to the low values of flux (as I_m is low and X_m is large) and rotor current compared with their rated values. The torque increases with the speed due to the increase in flux. A further increase in speed toward the positive slope of the characteristics increases the terminal voltage beyond the rated value. The flux and the magnetizing current are also increased, thereby saturating the flux. The torque can be controlled by the stator current and slip. To keep the air-gap flux constant and to avoid saturation due to high voltage, the motor is normally operated on the negative slope of the equivalent torque–speed characteristics with voltage control. The negative slope is in the unstable region and the motor must be operated in closed-loop control. At a low slip, the terminal voltage could be excessive and the flux would saturate. Due to saturation, the torque peaking, as shown in Figure 15.14a, is less than that as shown.

Figure 15.14b shows the steady-state torque–slip characteristic [23]. The maximum torque, when saturation is considered, becomes much smaller compared to the unsaturated case. The torque–speed characteristic is also shown for nominal stator voltages. This characteristic reflects operation at rated air-gap flux linkages.

The constant current can be supplied by three-phase current-source inverters. The current-fed inverter has the advantages of fault current control and the current

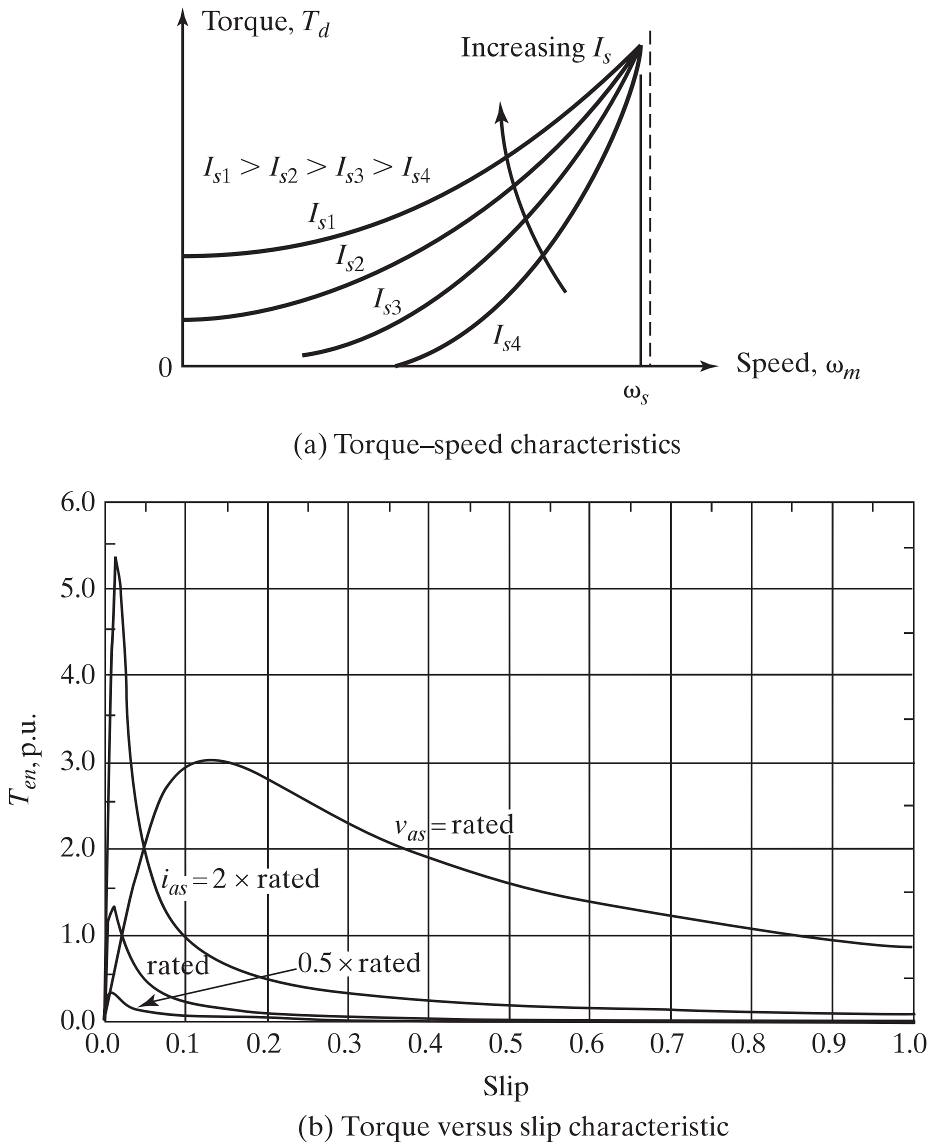


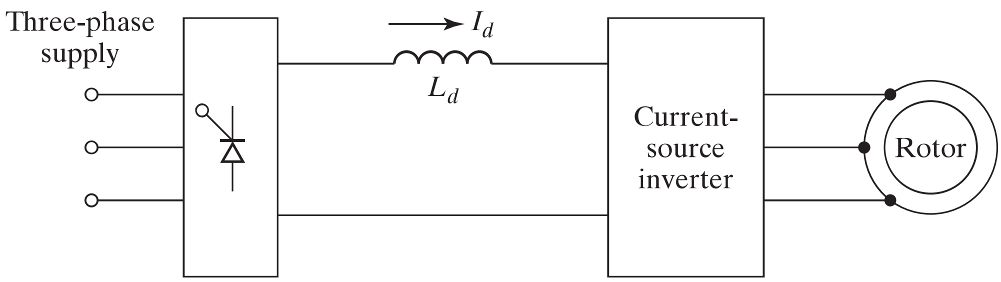
FIGURE 15.14

Torque–speed characteristics by current control.

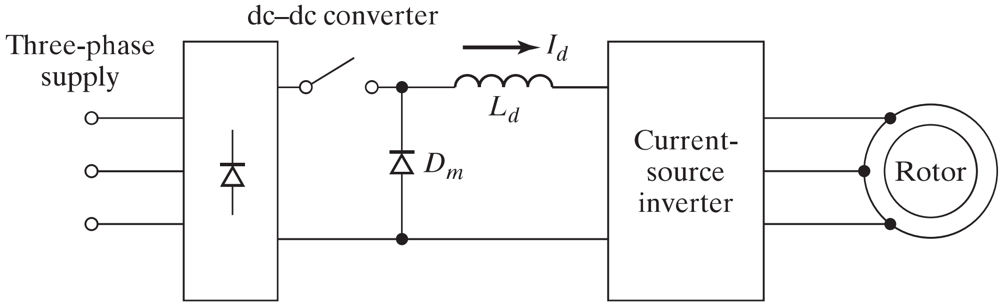
is less sensitive to the motor parameter variations. However, they generate harmonics and torque pulsation. Two possible configurations of current-fed inverter drives are shown in Figure 15.15. In Figure 15.15a, the inductor acts as a current source and the controlled rectifier controls the current source. The input PF of this arrangement is very low. In Figure 15.15b, the dc–dc converter controls the current source and the input PF is higher.

Example 15.7 Finding the Performance Parameters of a Three-Phase Induction Motor with Current Control

A three-phase, 11.2-kW, 1750-rpm, 460-V, 60-Hz, four-pole, Y-connected induction motor has the following parameters: $R_s = 0.66\ \Omega$, $R_r' = 0.38\ \Omega$, $X_s = 1.14\ \Omega$, $X_r' = 1.71\ \Omega$, and $X_m = 33.2\ \Omega$. The no-load loss is negligible. The motor is controlled by a current-source inverter and the input current is maintained constant at 20 A. If the frequency is 40 Hz and the developed



(a) Controlled rectifier-fed current source



(b) Chopper-fed current source

FIGURE 15.15

Current-source inductor motor drive.

torque is 55 N·m, determine (a) the slip for maximum torque s_m and maximum torque T_m, (b) the slip s, (c) the rotor speed ω_m, (d) the terminal voltage per phase V_a, and (e) the PF_m.

Solution

$V_{a(\text{rated})} = 460/\sqrt{3} = 265.58$ V, $I_i = 20$ A, $T_L = T_d = 55$ N·m, and $p = 4$. At 40 Hz, $\omega = 2\pi \times 40 = 251.33$ rad/s, $\omega_s = 2 \times 251.33/4 = 125.66$ rad/s, $R_s = 0.66\ \Omega$, $R'_r = 0.38\ \Omega$, $X_s = 1.14 \times 40/60 = 0.76\ \Omega$, $X'_r = 1.71 \times 40/60 = 1.14\ \Omega$, and $X_m = 33.2 \times 40/60 = 22.13\ \Omega$.

a. From Eq. (15.94),

$$s_m = \frac{0.38}{[0.66^2 + (22.13 + 0.78 + 1.14)^2]^{1/2}} = 0.0158$$

From Eq. (15.92) for $s = s_m$, $T_m = 94.68$ N·m.

b. From Eq. (15.92),

$$T_d = 55 = \frac{3(R_r/s)\,(22.13 \times 20)^2}{125.66 \times [(0.66 + R_r/s)^2 + (22.13 + 0.76 + 1.14)^2]}$$

which gives $(R_r/s)^2 - 83.74(R'_r/s) + 578.04 = 0$, and solving for R_r/s yields

$$\frac{R'_r}{s} = 76.144 \quad \text{or} \quad 7.581$$

and $s = 0.00499$ or 0.0501. Because the motor is normally operated with a large slip in the negative slope of the torque–speed characteristic,

$$s = 0.0501$$

c. $\omega_m = 125.656 \times (1 - 0.0501) = 119.36$ rad/s or 1140 rpm.

d. From Figure 15.2, the input impedance can be derived as

$$\overline{\mathbf{Z}}_i = R_i + jX_i = (R_i^2 + X_i^2)^{1/2}\angle\theta_m = Z_i\angle\theta_m$$

where

$$R_i = \frac{X_m^2(R_s + R_r/s)}{(R_s + R_r/s)^2 + (X_m + X_s + X_r)^2} \tag{15.98}$$

$$= 6.26\ \Omega$$

$$X_i = \frac{X_m[(R_s + R_r/s)^2 + (X_s + X_r)(X_m + X_s + X_r)]}{(R_s + R_r/s)^2 + (X_m + X_s + X_r)^2} \tag{15.99}$$

$$= 3.899\ \Omega$$

and

$$\theta_m = \tan^{-1}\frac{X_i}{R_i} \tag{15.100}$$

$$= 31.9°$$

$$Z_i = (6.26^2 + 3.899^2)^{1/2} = 7.38\ \Omega$$

$$V_a = Z_i I_i = 7.38 \times 20 = 147.6\ \text{V}$$

e. $\text{PF}_m = \cos(31.9°) = 0.849$ (lagging).

Note: If the maximum torque is calculated from Eq. (15.96), $T_m = 100.49$ and $V_a(\text{at } s = s_m)$ is 313 V. For a supply frequency of 90 Hz, recalculations of the values give $\omega_s = 282.74$ rad/s, $X_s = 1.71\ \Omega$, $X_r' = 2.565\ \Omega$, $X_m = 49.8\ \Omega$, $s_m = 0.00726$, $T_m = 96.1\ \text{N}\cdot\text{m}$, $s = 0.0225$, $V_a = 316$ V, and $V_a(\text{at } s = s_m) = 699.6$ V. It is evident that at a high frequency and a low slip, the terminal voltage would exceed the rated value and saturate the air-gap flux.

Example 15.8 Finding the Relationship between the Dc-Link Voltage and the Stator Frequency

The motor parameters of a volts/hertz inverter-fed induction motor drive are 6 hp, 220 V, 60 Hz, three phase, star connected, four poles, 0.86 PF and 84% efficiency, $R_s = 0.28$, $R_r = 0.17\Omega$, $X_m = 24.3\Omega$, $X_s = 0.56\Omega$, $X_r = 0.83\Omega$. Find (a) the maximum slip speed, (b) the rotor voltage drop V_o, (c) the volt–hertz constant K_{vf}, and (d) the dc-link voltage in terms of stator frequency f.

Solution

$HP = 6$ hp, $V_L = 220\ V$, $f = 60$ Hz, $p = 4$, $PF = 0.86$, $\eta_i = 84\%$, $R_s = 0.28$, $R_r = 0.17\Omega$, $X_m = 24.3\ \Omega$, $X_s = 0.56\ \Omega$, $X_r = 0.83\Omega$.

a. Using Eq. (15.25), the slip speed is

$$\omega_{sl} = \frac{R_r'}{X_s + R_r'}\omega = \frac{0.17}{0.56 + 0.83} \times 376.99 = 46.107\,\text{rad/s}$$

b. The stator phase current is given as

$$I_s = \frac{P_o}{3V_{ph} \times PF \times \eta_i} = \frac{4474}{3 \times 127 \times 0.86 \times 0.84} \times = 16.254\,\text{A}$$

$$V_o = I_s R_s = 16.254 \times 0.28 = 4.551\,\text{V}$$

c. Using Eq. (15.86), the volt-frequency constant is

$$K_{vf} = \frac{V_{ph} - V_o}{f} = \frac{127 - 4.551}{60} = 2.041\,\text{V/Hz}$$

d. Using Eq. (15.89), the dc voltage is

$$V_{\text{dc}} = \frac{V_o + K_{vf} f}{0.45} = 2.22 \times (4.551 + 2.041 f)$$

$$= 282.86\text{ V at } f = 60\text{ Hz}$$

15.2.8 Constant Slip-Speed Control

The slip speed ω_{sl} of the induction motor is maintained constant. That is, $\omega_{sl} = s\omega =$ constant. The slip is given by

$$s = \frac{\omega_{sl}}{\omega} = \frac{\omega_{sl}}{\omega_r + \omega_{sl}} \tag{15.101}$$

Thus, the slip $s = (\omega - \omega_r)/\omega$ will be varying with various rotor speeds $\omega_r = (p/2)\omega_m$ and the motor will operate in the normal torque-slip characteristic. Using the approximate equivalent circuit in Figure 15.2, the rotor current is given by

$$I_r = \frac{V_s}{\left(R_s + \dfrac{R_r'}{s}\right) + (X_s + X_r')} = \frac{V_s/\omega}{\left(R_s + \dfrac{R_r'}{\omega_{sl}}\right) + j(L_s + L_r')} \tag{15.102}$$

And the developed electromagnetic torque is

$$T_d = \frac{p}{2} \times \frac{P_d}{\omega} = 3 \times \frac{p}{2} \times \frac{I_r^2}{\omega}\left(\frac{R_r'}{s}\right) = 3 \times \frac{p}{2} \times \frac{I_r^2 R_r'}{\omega_{sl}} \tag{15.103}$$

Substituting the magnitude for I_r from Eq. (15.102) into Eq. (15.103) gives

$$T_d = 3 \times \frac{p}{2} \times \left(\frac{V_s}{\omega}\right)^2 \times \frac{\left(\dfrac{R_r'}{\omega_{sl}}\right)}{\left(R_s + \dfrac{R_r'}{\omega_{sl}}\right) + (L_s + L_r')^2} \tag{15.104}$$

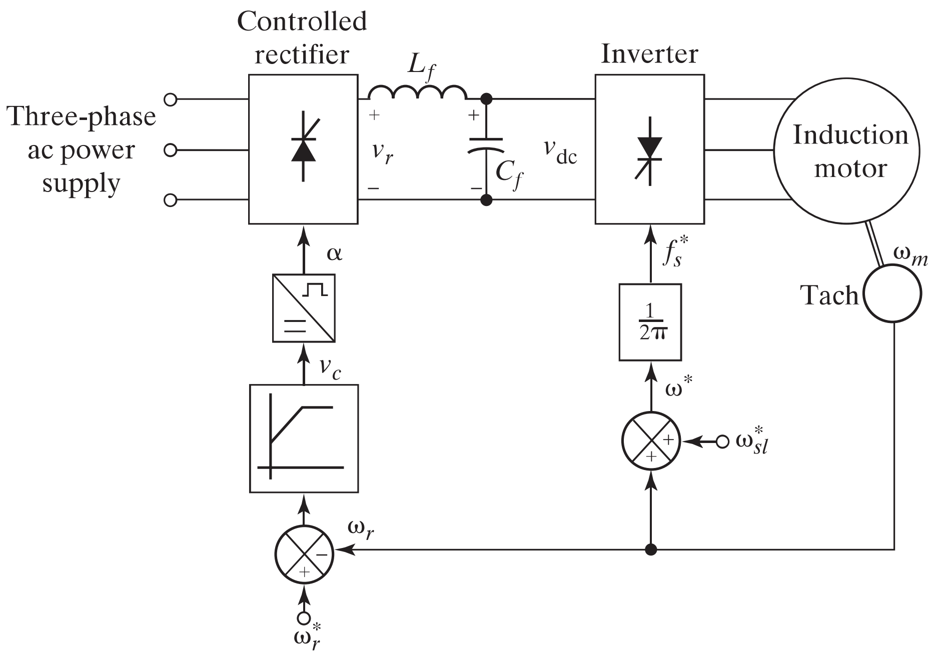


FIGURE 15.16

Block diagram for implementation of constant slip-speed control [23].

$$= K_{tc}\left(\frac{V_s}{\omega}\right)^2 \tag{15.105}$$

where torque constant K_{tc} is given by

$$K_{tc} = 3 \times \frac{p}{2} \times \frac{\left(\frac{R'_r}{\omega_{sl}}\right)}{\left(R_s + \frac{R'_r}{\omega_{sl}}\right) + (L_s + L'_r)^2} \tag{15.106}$$

According to Eq. (15.104), the torque depends on the square of the volts/hertz ratio and is independent of rotor speed $\omega_r = (p/2)\omega_m$. This type of control has the capability to produce a torque even at zero speed. This feature is essential in many applications, such as in robotics, where a starting or holding torque needs to be produced. The block diagram for the implementation of this control strategy is shown in Figure 15.16 [23]. The stator frequency is obtained by summing the slip speed ω_{sl}^* and the electrical rotor speed ω_r. The error speed signal is used to generate the delay angle α. A negative speed error clamps the bus voltage at zero, and triggering angles of greater than 900 are not allowed. This drive is restricted to one-quadrant operation only.

15.2.9 Voltage, Current, and Frequency Control

The torque–speed characteristics of induction motors depend on the type of control. It may be necessary to vary the voltage, frequency, and current to meet the torque–speed requirements, as shown in Figure 15.17, where there are three regions. In the first region, the speed can be varied by voltage (or current) control

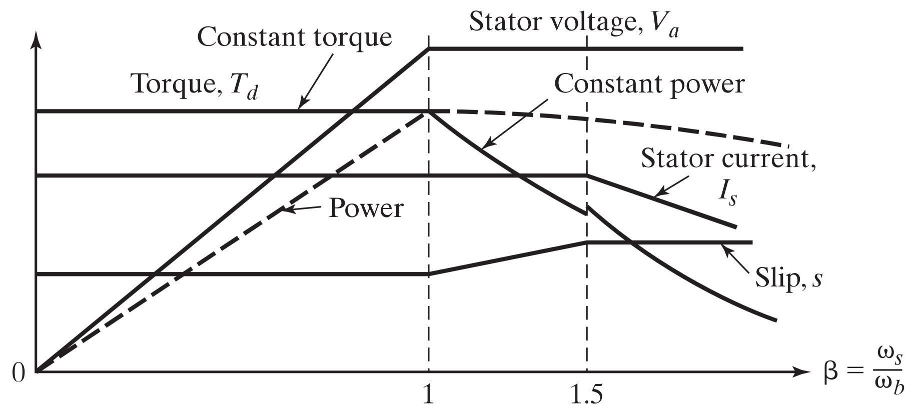


FIGURE 15.17
Control variables versus frequency.

at constant torque. In the second region, the motor is operated at constant current and the slip is varied. In the third region, the speed is controlled by frequency at a reduced stator current.

The torque and power variations for a given stator current and frequencies below the rated frequency are shown by dots in Figure 15.18. For $\beta < 1$, the motor operates at a constant flux. For $\beta > 1$, the motor is operated by frequency control, but at a constant voltage. Therefore, the flux decreases in the inverse ratio of per-unit frequency, and the motor operates in the field weakening mode.

When motoring, a decrease in speed command decreases the supply frequency. This shifts the operation to regenerative braking. The drive decelerates under the influence of braking torque and load torque. For speed below rate value ω_b, the voltage and frequency are reduced with speed to maintain the desired V to f ratio or constant flux and to keep the operation on the speed–torque curves with a negative slope by limiting the slip speed. For speed above ω_b, the frequency alone is reduced with the speed to maintain the operation on the portion of the speed–torque curves with a negative slope. When close to the desired speed, the operation shifts to motoring operation and the drive settles at the desired speed.

When motoring, an increase in the speed command increases the supply frequency. The motor torque exceeds the load torque and the motor accelerates. The operation is maintained on the portion of the speed–torque curves with a negative slope by limiting the slip speed. Finally, the drive settles at the desired speed.

Key Points of Section 15.2

- The speed and torque of induction motors can be varied by (1) stator voltage control; (2) rotor voltage control; (3) frequency control; (4) stator voltage and frequency control; (5) stator current control; or (6) voltage, current, and frequency control.
- To meet the torque–speed duty cycle of a drive, the voltage, current, and frequency are normally controlled such that the flux or the V to f ratio remains constant.

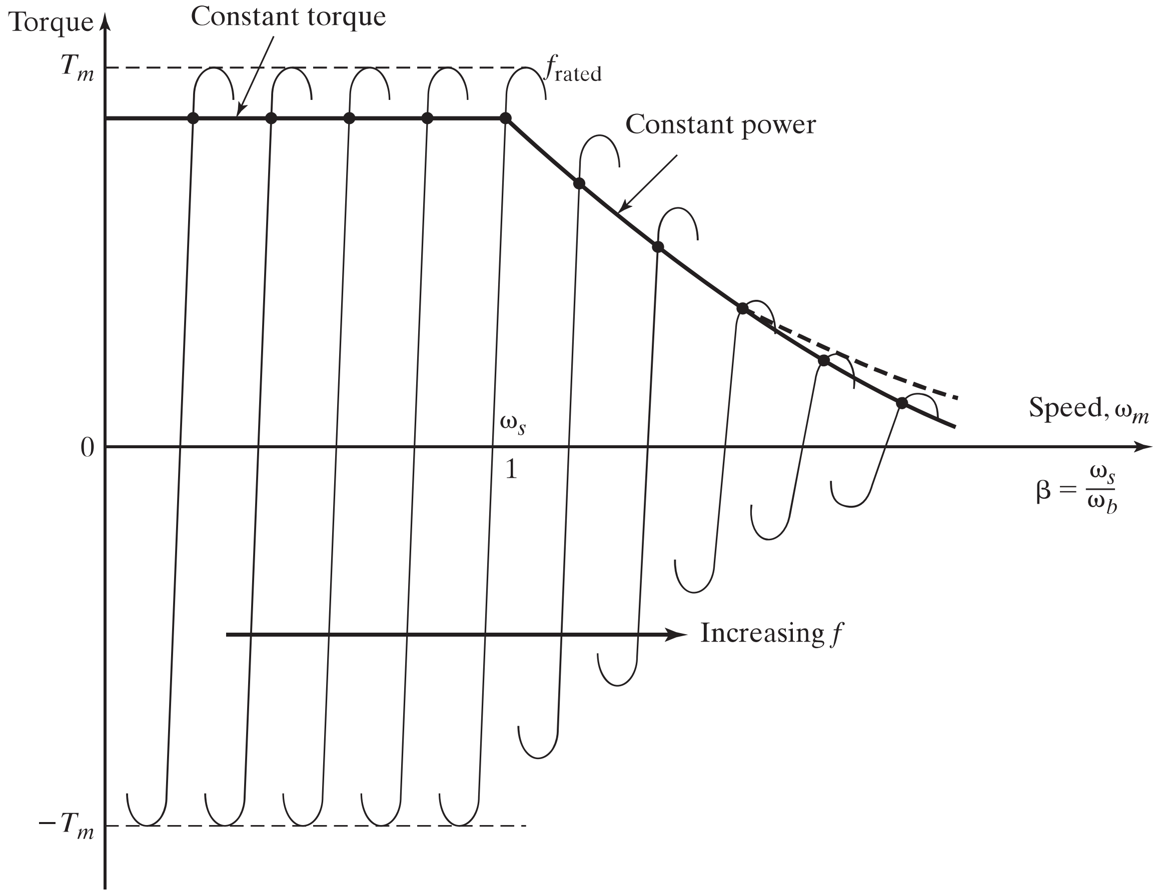


FIGURE 15.18

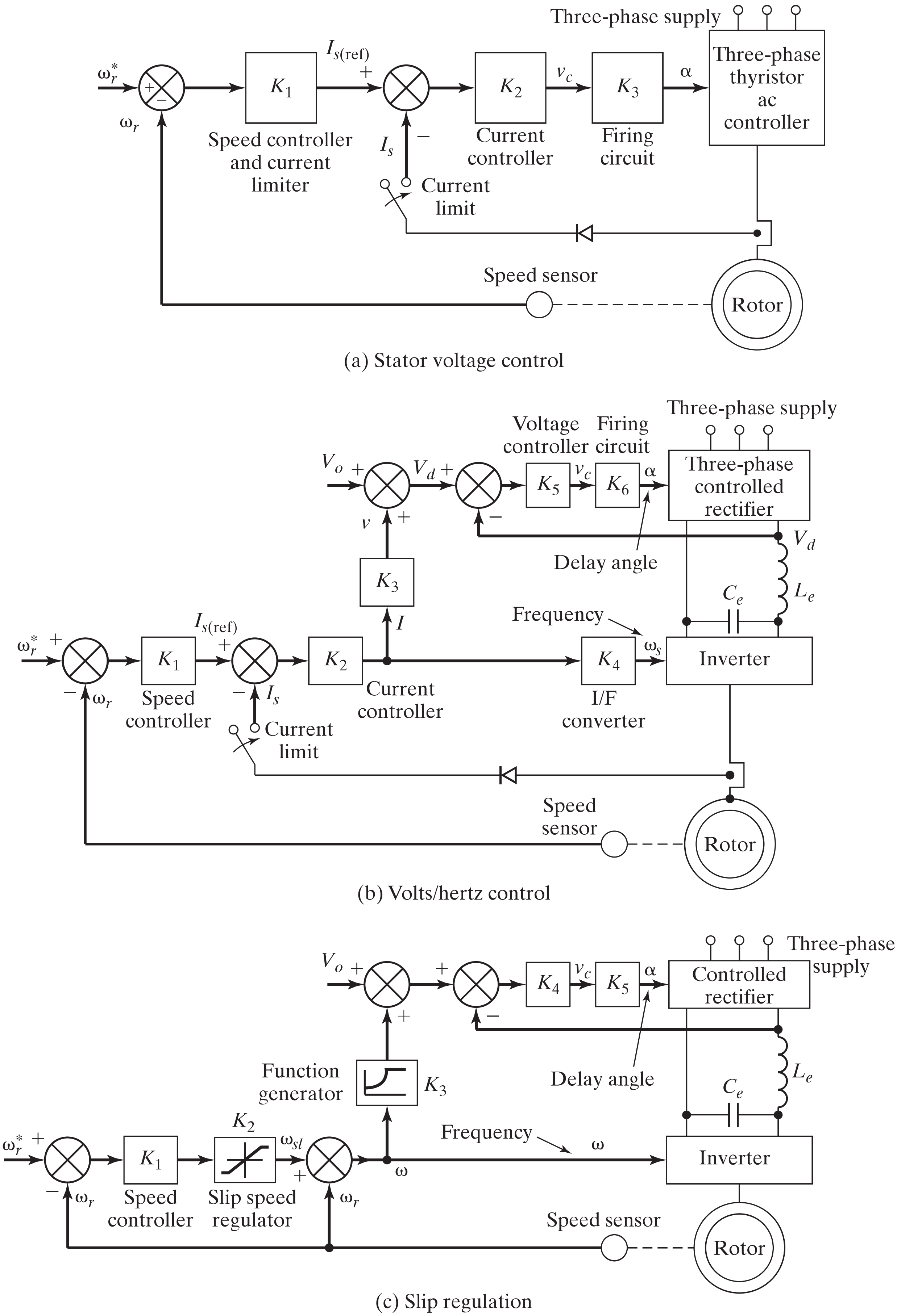
Torque–speed characteristics for variable frequency control.

15.3 CLOSED-LOOP CONTROL OF INDUCTION MOTORS

A closed-loop control is normally required to satisfy the steady-state and transient performance specifications of ac drives [9, 10]. The control strategy can be implemented by (1) *scalar control*, where the control variables are dc quantities and only their magnitudes are controlled; (2) *vector control*, where both the magnitude and phase of the control variables are controlled; or (3) *adaptive control*, where the parameters of the controller are continuously varied to adapt to the variations of the output variables.

The dynamic model of induction motors differs significantly from that of Figure 15.1c and is more complex than dc motors. The design of feedback-loop parameters requires complete analysis and simulation of the entire drive. The control and modeling of ac drives are beyond the scope of this book [2, 5, 17, 18]; only some of the basic scalar feedback techniques are discussed in this section.

A control system is generally characterized by the hierarchy of the control loops, where the outer loop controls the inner loops. The inner loops are designed to execute progressively faster. The loops are normally designed to have limited command excursion. Figure 15.19a shows an arrangement for stator voltage control of induction motors by ac voltage controllers at fixed frequency. The speed controller K_1 processes the speed error and generates the reference current $I_{s(\text{ref})}$. K_2 is the current controller. K_3 generates the delay angle of thyristor converter and the inner current-limit loop sets



(a) Stator voltage control

(b) Volts/hertz control

(c) Slip regulation

FIGURE 15.19

Closed-loop control of induction motors.

the torque limit indirectly. The current limiter instead of current clamping has the advantage of feeding back the short-circuit current in case of fault. The speed controller K_1 may be a simple gain (proportional type), proportional-integral type, or a lead-lag compensator. This type of control is characterized by poor dynamic and static performance and is generally used in fans, pumps, and blower drives.

The arrangement in Figure 15.19a can be extended to a volt/hertz control with the addition of a controlled rectifier and dc voltage control loop, as shown in Figure 15.19b. After the current limiter, the same signal generates the inverter frequency and provides input to the dc-link gain controller K_3. A small voltage V_0 is added to the dc voltage reference to compensate for the stator resistance drop at low frequency. The dc voltage V_d acts as the reference for the voltage control of the controlled rectifier. In case of PWM inverter, there is no need for the controlled rectifier and the signal V_d controls the inverter voltage directly by varying the modulation index. For current monitoring, it requires a sensor, which introduces a delay in the system response.

Because the torque of induction motors is proportional to the slip frequency, $\omega_{sl} = \omega_s - \omega_m = s\omega_s$, the slip frequency instead of the stator current can be controlled. The speed error generates the slip frequency command, as shown in Figure 15.19c, where the slip limits set the torque limits. The function generator, which produces the command signal for voltage control in response to the frequency ω_s, is nonlinear and also can take into account the compensating drop V_o at a low frequency. The compensating drop V_o is shown in Figure 15.19c. For a step change in the speed command, the motor accelerates or decelerates within the torque limits to a steady-state slip value corresponding to the load torque. This arrangement controls the torque indirectly within the speed control loop and do not require the current sensor.

A simple arrangement for current control is shown in Figure 15.20. The speed error generates the reference signal for the dc-link current. The slip frequency, $\omega_{sl} = \omega - \omega_r$, is fixed. With a step speed command, the machine accelerates with a high current that is proportional to the torque. In the steady state, the motor current is low. However, the air-gap flux fluctuates, and due to varying flux at different operating points, the performance of this drive is poor.

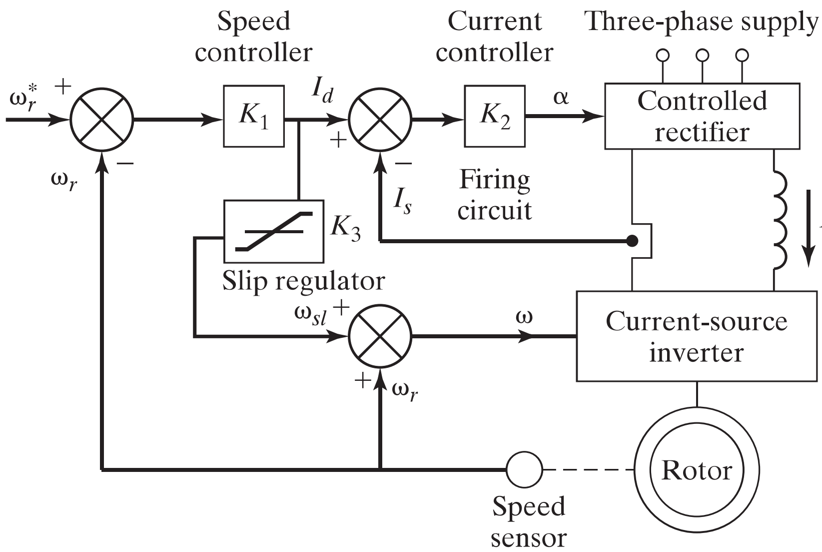


FIGURE 15.20

Current control with constant slip.

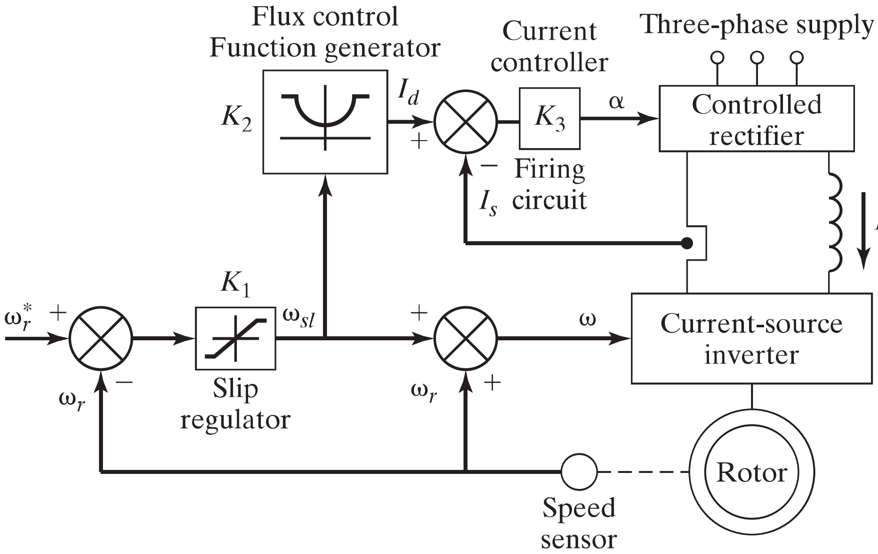


FIGURE 15.21

Current control with constant flux operation.

A practical arrangement for current control, where the flux is maintained constant, is shown in Figure 15.21. The speed error generates the slip frequency, which controls the inverter frequency and the dc-link current source. The function generator produces the current command to maintain the air-gap flux constant, normally at the rated value.

The arrangement in Figure 15.19a for speed control with inner current control loop can be applied to a static Kramer drive, as shown in Figure 15.22, where the torque is proportional to the dc-link current I_d. The speed error generates the dc-link current command. A step increase in speed clamps the current to the maximum value and the motor accelerates at a constant torque that corresponds to the maximum current. A step decrease in the speed sets the current command to zero and the motor decelerates due to the load torque.

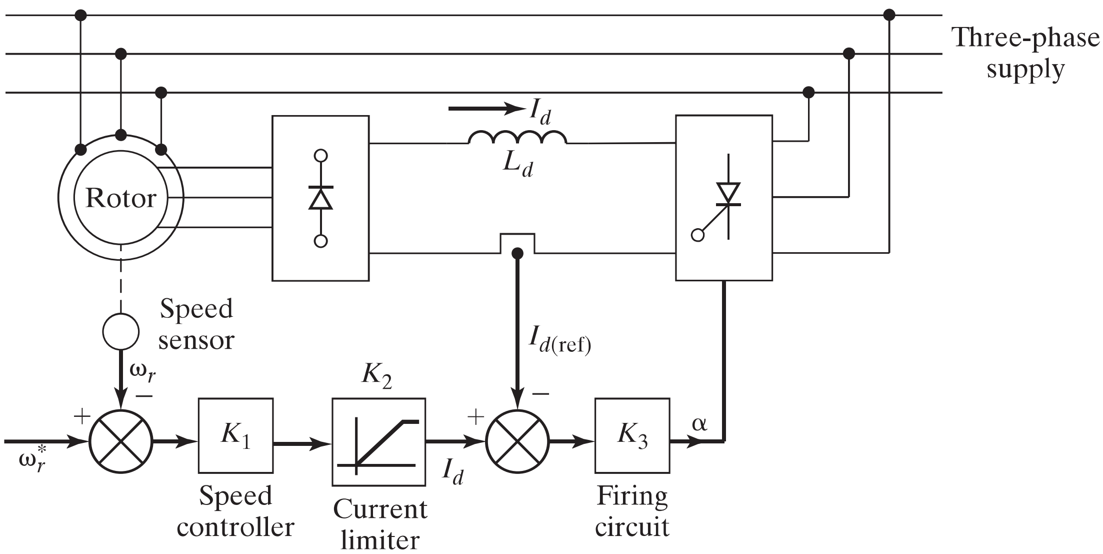


FIGURE 15.22

Speed control of static Kramer drive.

Key Points of Section 15.3

- The closed-loop is normally used to control the steady-state and transient response of ac drives.
- However, the parameters of induction motors are coupled to each other and the scalar control lacks in producing fast dynamic response.

15.4 DIMENSIONING THE CONTROL VARIABLES

The control variables in Figures 15.19 to 15.22 show the relationship between the input and outputs of control blocks with gain contents. Example 15.8 illustrates the relationship of the dc-link voltage V_{dc} to the stator frequency f. For practical implementation, these variables and the constants must be scaled to the control signal levels. Figure 15.23 shows the block diagram of the volts/hertz-controlled induction motor drive [23].

The external signal v^* is generated from the speed command ω_r^* and scaled by a proportionality constant K^* as given by

$$K^* = \frac{v^*}{\omega_r^*} = \frac{V_{cm}}{\omega_{r(\max)}^*} \tag{15.107}$$

where V_{cm} is the maximum control signal and its value is usually in the range of $\pm 10\,\text{V}$ or $\pm 5\text{V}$. The range of v^* is given by

$$-V_{cm} < v^* < V_{cm} \tag{15.108}$$

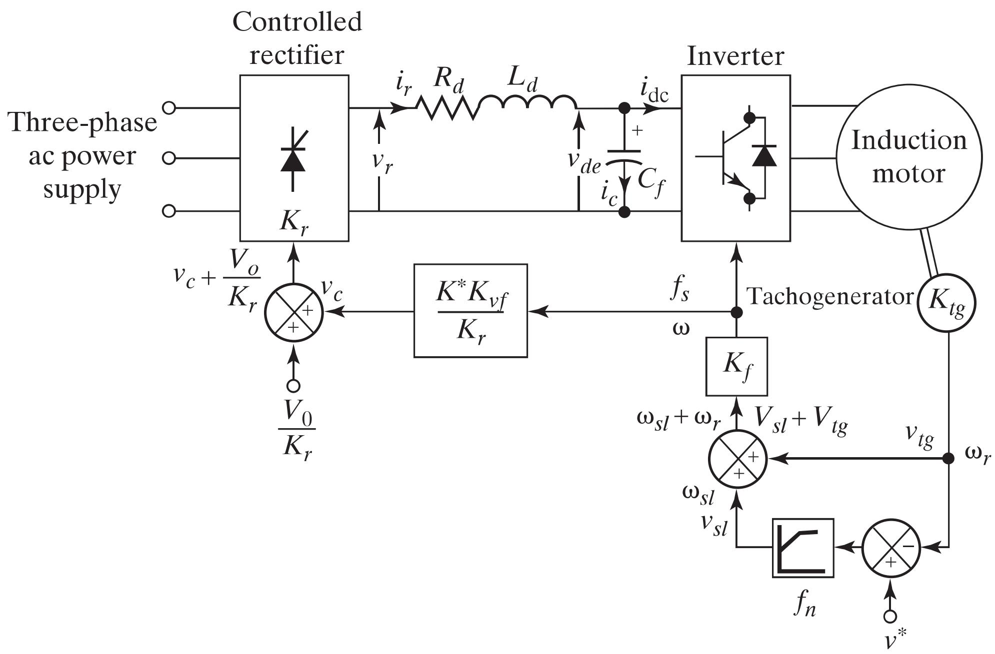


FIGURE 15.23

Block diagram of the volts/hertz-controlled induction motor drive [23].

The gain of the tachogenerator block is adjusted to have its maximum output corresponding to $\pm V_{cm}$ for control compatibility. Thus, the gain of the tachogenerator and the filter is given by

$$K_{tg} = \frac{V_{cm}}{\omega_r (p/2)} = \frac{p}{2} K^* \tag{15.109}$$

The maximum slip speed corresponds to the maximum torque of the induction motor and the corresponding slip voltage is given by

$$v_{sl(\max)} = K^* \omega_{sl(\max)} \tag{15.110}$$

The sum of the slip-speed signal and the rotor electrical speed signal corresponds to the supply speed. That is, $\omega_{sl} + \omega_r = \omega$. Therefore, the gain of the frequency transfer block is

$$K_f = \frac{1}{2\pi K^*} \tag{15.111}$$

Using Eqs. (15.109) and (15.110), the stator frequency f is given by

$$f = K_f\left(K^* \omega_{sl} + \frac{p}{2} K^* \omega_m\right) = K_f K^* (\omega_{sl} + \omega_r) \tag{15.112}$$

Using Eq. (15.89), the control voltage of the output rectifier is given by

$$v_c = \frac{1}{0.45 K_r}(V_o + K_{vf} f) = \frac{2.22}{K_r}(V_o + K_{vf} f) \tag{15.113}$$

where K_r is the gain of the controlled rectifier. The output of the rectifier is given by

$$v_r = K_r v_c = 2.22 \times (V_o + K_{vf} f) = 2.22 \times [V_o + K_{vf} K_f K^* (\omega_{sl} + \omega_r)] \tag{15.114}$$

Using Eq. (15.112), the supply speed ω is given by

$$\omega = 2\pi f = 2\pi K_f K^* (\omega_{sl} + \omega_r) \tag{15.115}$$

and the slip speed is given by

$$\omega_{sl} = f_{sc}(v^* - v_{tg}) = f_{sc}(v^* - \omega_m K_{tg}) = f_{sc}(K^* \omega_r^* - \omega_m K_{tg}) \tag{15.116}$$

where f_{sc} is the speed controller function.

Example 15.9 Finding the Dimension Factors of the Control Variables

For the induction motor in Example 15.8, find (a) the constants K^*, K_{tg}, K_f and (b) express the rectifier output voltage v_r in terms of slip frequency ω_{sl} if the rated mechanical speed is $N = 1760\,\text{rpm}$ and $V_{cm} = 10\,\text{V}$.

Solution

$p = 4, N = 1760$ rpm, $\omega_m = (2\pi N)/60 = (2\pi \times 1760)/60 = 157.08$ rad/s, $\omega_r = (p/2) \times \omega_m = (4/2) \times 157.08 = 314.159$ rad/s, $\omega_{r(max)} = \omega_r = 314.159$ rad/s.

From Example 15.8, $K_{vf} = 4.551$ V/Hz

a. Using Eq. (15.107), $K^* = V_{cm}/\omega_{r(\max)} = 10/314.159 = 0.027$ V/rad/s
Using Eq. (15.109), $K_{tg} = (p/2)K^* = (4/2) \times 0.027 = 0.053$ V/rad/s
Using Eq. (15.111), $K_f = 1/(2\pi K^*) = 1/(2 \times \pi \times 0.027) = 6$ Hz/rad/s

b. Using Eq. (15.114), the rectifier output voltage is

$$v_r = 2.22 \times [V_o + K_{vf}K_fK^*(\omega_{sl} + \omega_r)]$$

$$= 2.22 \times [4.551 + 2.041 \times 6 \times 0.027 \times (\omega_{sl} + \omega_r)]$$

$$= 10.103 + 0.721 \times (\omega_{sl} + \omega_r)$$

15.5 VECTOR CONTROLS

The control methods that have been discussed so far provide satisfactory steady-state performance, but their dynamic response is poor. An induction motor exhibits nonlinear multivariables and highly coupled characteristics. The *vector control* technique, which is also known as *field-oriented control (FOC)*, allows a squirrel-cage induction motor to be driven with high dynamic performance that is comparable to the characteristic of a dc motor [11–15]. The FOC technique decouples the two components of stator current: one providing the air-gap flux and the other producing the torque. It provides independent control of flux and torque, and the control characteristic is linearized. The stator currents are converted to a fictitious synchronously rotating reference frame aligned with the flux vector and are transformed back to the stator frame before feeding back to the machine. The two components are d-axis i_{ds} analogous to field current and q-axis i_{qs} analogous to the armature current of a separately excited dc motor. The rotor flux linkage vector is aligned along the d-axis of the reference frame.

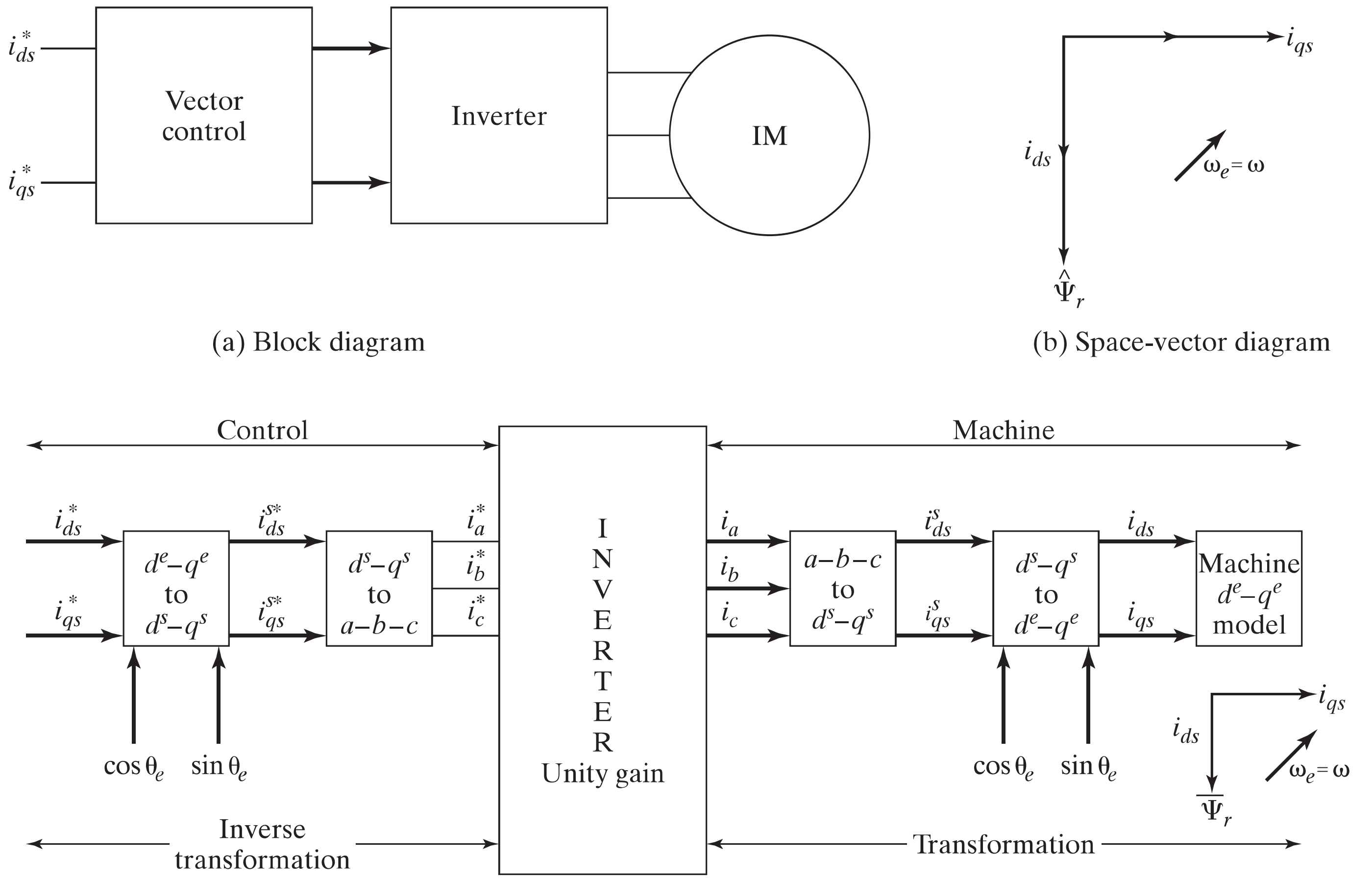
15.5.1 Basic Principle of Vector Control

With a vector control, an induction motor can operate as a separately excited dc motor. In a dc machine, the developed torque is given by

$$T_d = K_t I_a I_f \tag{15.117}$$

where I_a is the armature current and I_f is the field current. The construction of a dc machine is such that the field flux linkage Ψ_f produced by I_f is perpendicular to the armature flux linkage Ψ_a produced by I_a. These flux vectors that are stationary in space are orthogonal or decoupled in nature. As a result, a dc motor has fast transient response. However, an induction motor cannot give such fast response due to its inherent coupling problem. However, an induction motor can exhibit the dc machine characteristic if the machine is controlled in a synchronously rotating frame $(d^e - q^e)$, where the sinusoidal machine variables appear as dc quantities in the steady state.

Figure 15.24a shows an inverter-fed induction motor with two control current inputs: i_{ds}^* and i_{qs}^* are the direct-axis component and quadrature-axis component of



(a) Block diagram

(b) Space-vector diagram

(c) Vector control implementation

FIGURE 15.24

Vector control of induction motor.

the stator current, respectively, in a synchronously rotating reference frame. With vector control, i_{qs}^* is analogous to the field current I_f and i_{ds}^* is analogous to armature current I_a of a dc motor. Therefore, the developed torque of an induction motor is given by

$$T_d = K_m \hat{\Psi}_r I_f = K_t I_{ds} i_{qs_r} \tag{15.118}$$

where $\hat{\Psi}_r$ is the absolute peak value of the sinusoidal space flux linkage vector $\vec{\Psi}_r$;
i_{qs} is the field component;
i_{ds} is the torque component.

Figure 15.24b shows the space-vector diagram for vector control: i_{ds}^* is oriented (or aligned) in the direction of rotor flux $\hat{\lambda}_r$, and i_{qs}^* must be perpendicular to it under all operating conditions. The space vectors rotate synchronously at electrical frequency $\omega_e = \omega$. Thus, the vector control must ensure the correct orientation of the space vectors and generate the control input signals.

The implementation of the vector control is shown in Figure 15.24c. The inverter generates currents i_a, i_b, and i_c in response to the corresponding command currents i_a^*, i_b^*, and i_c^* from the controller. The machine terminal currents i_a, i_b, and i_c are converted to i_{ds}^s and i_{qs}^s components by three-phase to two-phase transformation. These are then converted to synchronously rotating frame (into i_{ds} and i_{qs} components) by the unit vector components $\cos\theta_e$ and $\sin\theta_e$ before applying them to the machine. The machine is represented by internal conversions into the $d^e - q^e$ model.

The controller makes two stages of inverse transformation so that the line control currents i_{ds}^* and i_{qs}^* correspond to the machine currents i_{ds} and i_{qs}, respectively. In addition, the unit vector ($\cos\theta_e$ and $\sin\theta_e$) ensures correct alignment of i_{ds} current with the flux vector Ψ_r and i_{qs} current is perpendicular to it. It is important to note that the transformation and inverse transformation ideally do not incorporate any dynamics. Therefore, the response to i_{ds} and i_{qs} is instantaneous except for any delays due to computational and sampling times.

15.5.2 Direct and Quadrature-Axis Transformation

The vector control technique uses the dynamic equivalent circuit of the induction motor. There are at least three fluxes (rotor, air gap, and stator) and three currents or mmfs (in stator, rotor, and magnetizing) in an induction motor. For fast dynamic response, the interactions between current, fluxes, and speed must be taken into account in obtaining the dynamic model of the motor and determining appropriate control strategies.

All fluxes rotate at synchronous speed. The three-phase currents create mmfs (stator and rotor), which also rotate at synchronous speed. Vector control aligns axes of an mmf and a flux orthogonally at all times. It is easier to align the stator current mmf orthogonally to the rotor flux.

Any three-phase sinusoidal set of quantities in the stator can be transformed to an orthogonal reference frame by

$$\begin{bmatrix} f_{\alpha s} \\ f_{\beta s} \\ f_o \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{4\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{4\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} f_{as} \\ f_{bs} \\ f_{cs} \end{bmatrix} \tag{15.119}$$

where θ is the angle of the orthogonal set α-β-0 with respect to any arbitrary reference. If the α-β-0 axes are stationary and the α-axis is aligned with the stator a-axis, then $\theta = 0$ at all times. Thus, we get

$$\begin{bmatrix} f_{\alpha s} \\ f_{\beta s} \\ f_{os} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} f_{as} \\ f_{bs} \\ f_{cs} \end{bmatrix} \tag{15.120}$$

The orthogonal set of reference that rotates at the synchronous speed ω is referred to as d–q–0 axes. Figure 15.25 shows the axis of rotation for various quantities.

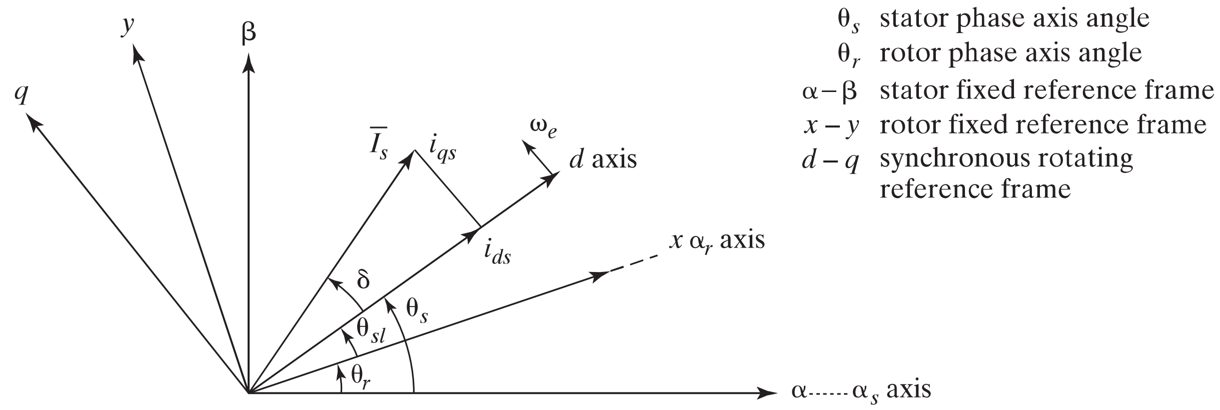


FIGURE 15.25

Axis of rotation for various quantities.

The three-phase rotor variables, transformed to the synchronously rotating frame, are given by

$$\begin{bmatrix} f_{dr} \\ f_{qr} \\ f_o \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos(\omega - \omega_r)t & \cos\left((\omega - \omega_r)t - \frac{2\pi}{3}\right) & \cos\left((\omega - \omega_r)t - \frac{4\pi}{3}\right) \\ \sin(\omega - \omega_r)t & \sin\left((\omega - \omega_r)t - \frac{2\pi}{3}\right) & \sin\left((\omega - \omega_r)t - \frac{4\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} f_{ar} \\ f_{br} \\ f_{cr} \end{bmatrix} \tag{15.121}$$

where the slip is defined by Eq. (15.4a).

It is important to note that the difference $\omega - \omega_r$ is the relative speed between the synchronously rotating reference frame and the frame attached to the rotor. This difference is slip frequency ω_{sl}, which is the frequency of the rotor variables. By applying these transformations, voltage equations of the motor in the synchronously rotating frame are reduced to,

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{qr} \\ v_{dr} \end{bmatrix} = \begin{bmatrix} R_s + DL_s & \omega L_s & DL_m & \omega L_m \\ -\omega L_s & R_s + DL_s & -\omega L_m & DL_m \\ DL_m & (\omega - \omega_r)L_m & R_r + DL_r & (\omega - \omega_r)L_r \\ -(\omega - \omega_r)L_m & DL_m & -(\omega - \omega_r)L_r & R_r + DL_r \end{bmatrix}\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} \tag{15.122}$$

where ω is the speed of the reference frame (or synchronous speed), ω_r is rotor speed, and

$$L_s = L_{ls} + L_m, \; L_r = L_{lr} + L_m$$

Subscripts *l* and *m* stand for leakage and magnetizing, respectively, and *D* represents the differential operator *d/dt*. The dynamic equivalent circuits of the motor in this reference frame are shown in Figure 15.26.

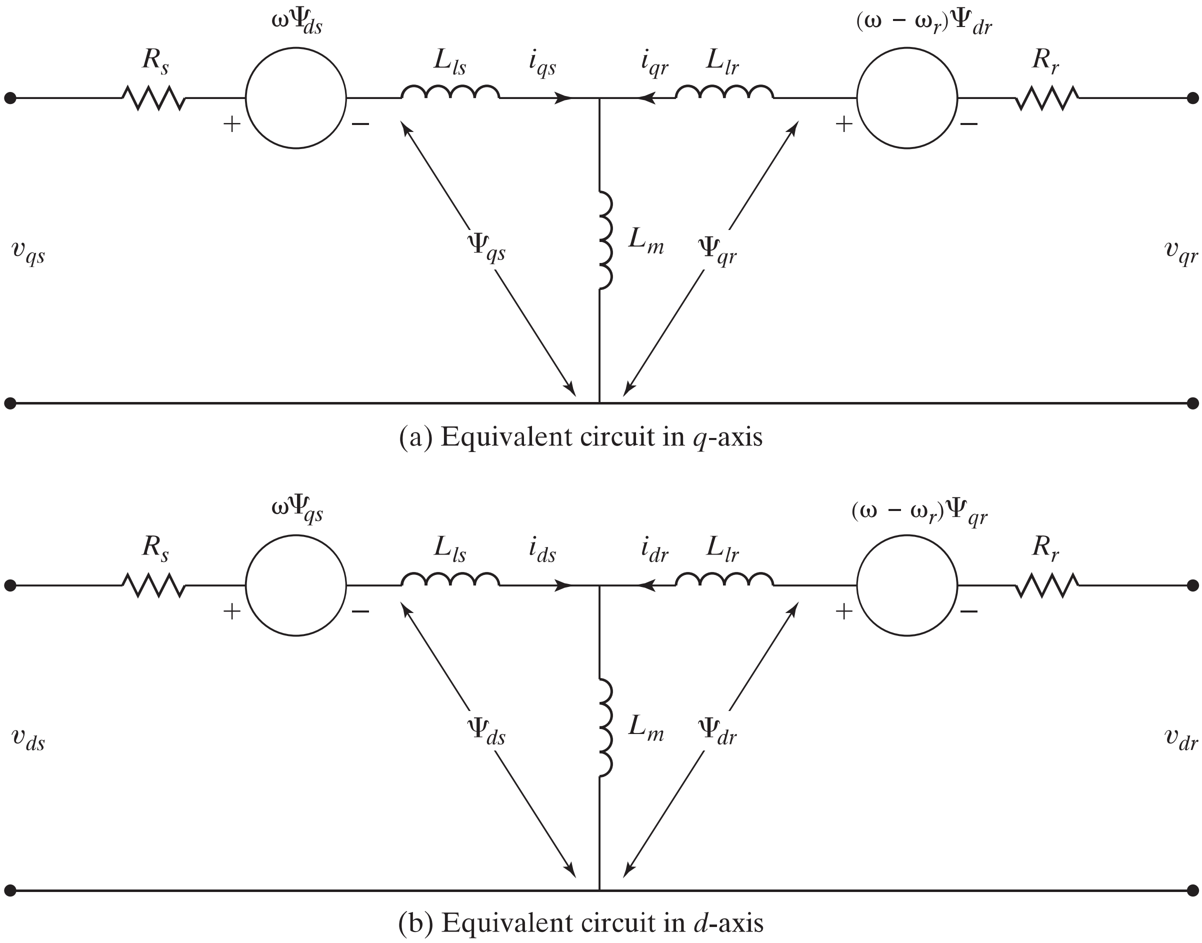


FIGURE 15.26

Dynamic equivalent circuits in the synchronously rotating frame.

The stator flux linkages are expressed as

$$\Psi_{qs} = L_{ls} i_{qs} + L_m (i_{qs} + i_{qr}) = L_s i_{qs} + L_m i_{qr} \tag{15.123}$$

$$\Psi_{ds} = L_{ls} i_{ds} + L_m (i_{ds} + i_{dr}) = L_s i_{ds} + L_m i_{dr} \tag{15.124}$$

$$\hat{\Psi}_s = \sqrt{(\Psi_{qs}^2 + \Psi_{ds}^2)} \tag{15.125}$$

The rotor flux linkages are given by

$$\Psi_{qr} = L_{lr} i_{qr} + L_m (i_{qs} + i_{qr}) = L_r i_{qr} + L_m i_{qs} \tag{15.126}$$

$$\Psi_{dr} = L_{lr} i_{dr} + L_m (i_{ds} + i_{dr}) = L_r i_{dr} + L_m i_{ds} \tag{15.127}$$

$$\hat{\Psi}_r = \sqrt{(\Psi_{qr}^2 + \Psi_{dr}^2)} \tag{15.128}$$

The air-gap flux linkages are given by

$$\Psi_{mq} = L_m (i_{qs} + i_{qr}) \tag{15.129}$$

$$\Psi_{md} = L_m (i_{ds} + i_{dr}) \tag{15.130}$$

$$\hat{\Psi}_m = \sqrt{(\Psi_{mqs}^2 + \Psi_{mds}^2)} \tag{15.131}$$

Therefore, the torque developed by the motor is given by

$$T_d = \frac{3p}{2}\frac{[\Psi_{ds}i_{qs} - \Psi_{qs}i_{ds}]}{2} \tag{15.132}$$

where p is the number of poles. Equation (15.122) gives the rotor voltages in d- and q-axis as

$$v_{qr} = 0 = L_m\frac{di_{qs}}{dt} + (\omega - \omega_r)L_m i_{ds} + (R_r + L_r)\frac{di_{qr}}{dt} + (\omega - \omega_r)L_r i_{dr} \tag{15.133}$$

$$v_{dr} = 0 = L_m\frac{di_{ds}}{dt} + (\omega - \omega_r)L_m i_{qs} + (R_r + L_r)\frac{di_{dr}}{dt} + (\omega - \omega_r)L_r i_{qr} \tag{15.134}$$

which, after substituting Ψ_{qr} from Eq. (15.126) and Ψ_{dr} from Eq. (15.127), gives

$$\frac{d\Psi_{qr}}{dt} + R_r i_{qr} + (\omega - \omega_r)\Psi_{dr} = 0 \tag{15.135}$$

$$\frac{d\Psi_{dr}}{dt} + R_r i_{dr} + (\omega - \omega_r)\Psi_{qr} = 0 \tag{15.136}$$

Solving for i_{qr} from Eq. (15.126) and i_{dr} from Eq. (15.127), we get

$$i_{qr} = \frac{1}{L_r}\Psi_{qr} - \frac{L_m}{L_r}i_{qs} \tag{15.137}$$

$$i_{dr} = \frac{1}{L_r}\Psi_{dr} - \frac{L_m}{L_r}i_{ds} \tag{15.138}$$

Substituting these rotor currents i_{qr} and i_{dr} into Eqs. (15.135) and (15.136), we get

$$\frac{d\Psi_{qr}}{dt} + \frac{L_r}{R_r}\Psi_{qr} - \frac{L_m}{L_r}R_r i_{qs} + (\omega - \omega_r)\Psi_{dr} = 0 \tag{15.139}$$

$$\frac{d\Psi_{qr}}{dt} + \frac{L_r}{R_r}\Psi_{qr} - \frac{L_m}{L_r}R_r i_{qs} + (\omega - \omega_r)\Psi_{dr} = 0 \tag{15.140}$$

To eliminate transients in the rotor flux and the coupling between the two axes, the following conditions must be satisfied.

$$\Psi_{qr} = 0 \text{ and } \hat{\Psi}_r = \sqrt{\Psi_{dr}^2 + \Psi_{qr}^2} = \Psi_{dr} \tag{15.141}$$

Also, the rotor flux should remain constant so that

$$\frac{d\Psi_{dr}}{dt} = \frac{d\Psi_{qr}}{dt} = 0 \tag{15.142}$$

With conditions in Eqs. (15.141) and (15.142), the rotor flux $\hat{\Psi}_r$ is aligned on the d^e-axis, and we get

$$\omega - \omega_r = \omega_{sl} = \frac{L_m}{\hat{\Psi}_r}\frac{R_r}{L_r}i_{qs} \tag{15.143}$$

and

$$\frac{L_r}{R_r}\frac{d\hat{\Psi}_r}{dt} + \hat{\Psi}_r = L_m i_{ds} \tag{15.144}$$

Substituting the expressions for i_{qr} from Eq. (15.137) into Eq. (15.126) and i_{dr} from Eq. (15.138) into Eq. (15.127), we get

$$\Psi_{qs} = \left(L_s - \frac{L_m^2}{L_r}\right) i_{qs} + \frac{L_m}{L_r}\Psi_{qr} \tag{15.145}$$

$$\Psi_{ds} = \left(L_s - \frac{L_m^2}{L_r}\right) i_{ds} + \frac{L_m}{L_r}\Psi_{dr} \tag{15.146}$$

Substituting Ψ_{qs} Eq. (15.145) and Ψ_{ds} from Eq. (15.146) into Eq. (15.132) gives the developed torque as

$$T_d = \frac{3p}{2}\frac{L_m}{L_r}\left(\frac{\Psi_{dr} i_{qs} - \Psi_{qr} i_{ds}}{2}\right) = \frac{3p}{2 \times 2}\frac{L_m}{L_r}\hat{\Psi}_r i_{qs} \tag{15.147}$$

If the rotor flux $\hat{\Psi}_r$ remains constant, Eq. (15.144) becomes

$$\hat{\Psi}_r = L_m i_{ds} \tag{15.148}$$

which indicates that the rotor flux is directly proportional to current i_{ds}. Thus, T_d becomes

$$T_d = \frac{3p}{2 \times 2}\frac{L_m^2}{L_r} i_{ds} i_{qs} = K_m i_{ds} i_{qs} \tag{15.149}$$

where $K_m = 3pL_m^2/4L_r$.

The vector control can be implemented by either direct method or indirect method [4]. The methods are different essentially by how the unit vector ($\cos\theta_s$ and $\sin\theta_s$) is generated for the control. In the direct method, the flux vector is computed from the terminal quantities of the motor, as shown in Figure 15.27a. The indirect method uses the motor slip frequency ω_{sl} to compute the desired flux vector, as shown in Figure 15.27b. It is simpler to implement than the direct method and is used increasingly in induction motor control. T_d is the desired motor torque, Ψ_r is the rotor flux linkage, T_r is the rotor time constant, and L_m is the mutual inductance. The amount of decoupling is dependent on the motor parameters unless the flux is measured directly. Without the exact knowledge of the motor parameters, an ideal decoupling is not possible.

Notes:

1. According to Eq. (15.144), the rotor flux $\hat{\Psi}_r$ is determined by i_{dr}, which is subject to a time delay T_r due to the rotor time constant (L_r/R_r).
2. According to Eq. (15.149), the current i_{qs} controls the developed torque T_d without delay.
3. Currents i_{ds} and i_{qs} are orthogonal to each other and are called the *flux*- and *torque*-producing currents, respectively. This correspondence between flux- and

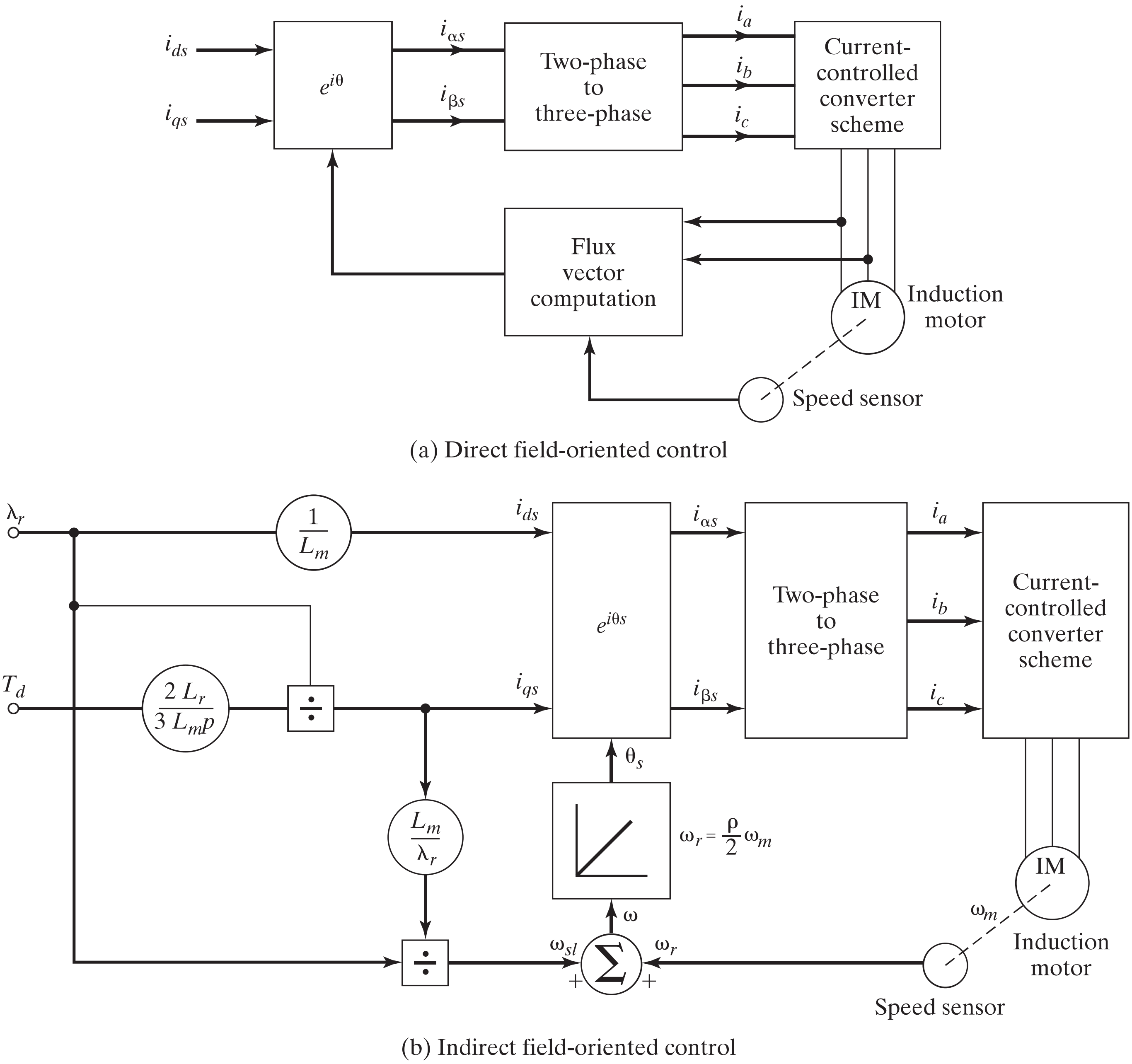


FIGURE 15.27

Block diagrams of vector control.

torque-producing currents is subject to maintaining the conditions in Eqs. (15.143) and (15.144). Normally, i_{ds} would remain fixed for operation up to the base speed. Thereafter, it is reduced to weaken the rotor flux so that the motor may be driven with a constant power like characteristic.

15.5.3 Indirect Vector Control

Figure 15.28 shows the block diagram for control implementation of the indirect field-oriented control (IFOC). The flux component of current i_{ds}^* for the desired rotor flux $\hat{\Psi}_r$ is determined from Eq. (15.148), and is maintained constant. The variation

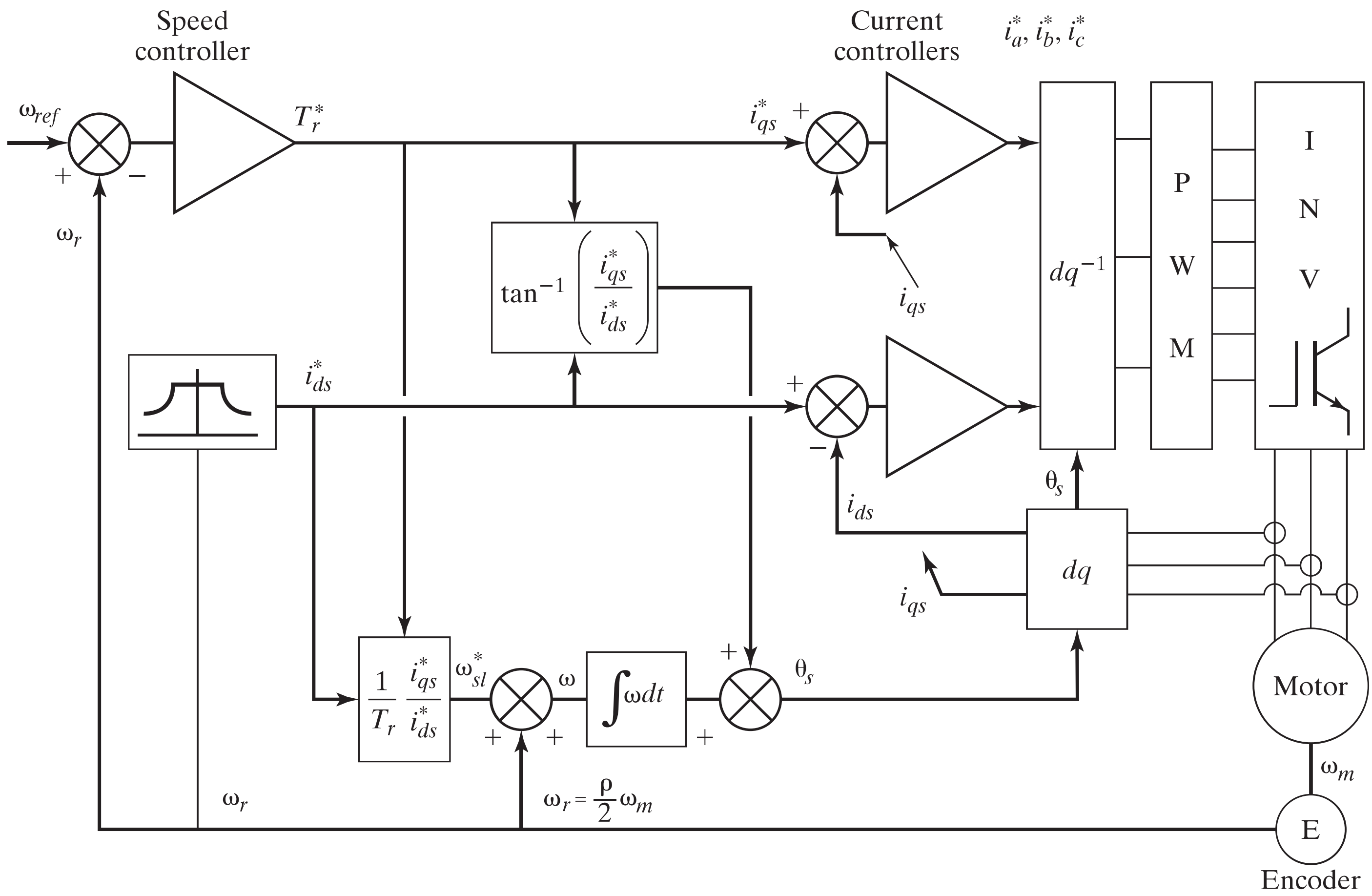


FIGURE 15.28

Indirect rotor flux–oriented control scheme.

of magnetizing inductance L_m can, however, cause some drift in the flux. Equation (15.143) relates the angular speed error $(\omega_{\text{ref}} - \omega_r)$ to i_{qs}^* by

$$i_{qs}^* = \frac{\hat{\Psi}_r L_r}{L_m R_r}(\omega_{\text{ref}} - \omega_r) \tag{15.150}$$

which, in turn, generates the torque component of current i_{qs}^* from the speed control loop. The slip frequency ω_{sl}^* is generated from i_{qs}^* in feed-forward manner from Eq. (15.143). The corresponding expression of slip gain K_{sl} is given by

$$K_{sl} = \frac{\omega_{sl}^*}{i_{qs}^*} = \frac{L_m}{\hat{\Psi}_r} \times \frac{R_r}{L_r} = \frac{R_r}{L_r} \times \frac{1}{i_{ds}^*} \tag{15.151}$$

The slip speed ω_{sl}^* is added to the rotor speed ω_r to obtain the stator frequency ω. This frequency is integrated with respect to time to produce the required angle θ_s of the stator mmf relative to the rotor flux vector. This angle is used to generate the unit vector signals ($\cos\theta_s$ and $\sin\theta_s$) and to transform the stator currents (i_{ds} and i_{qs}) to the dq reference frame. Two independent current controllers are used to regulate the i_q and

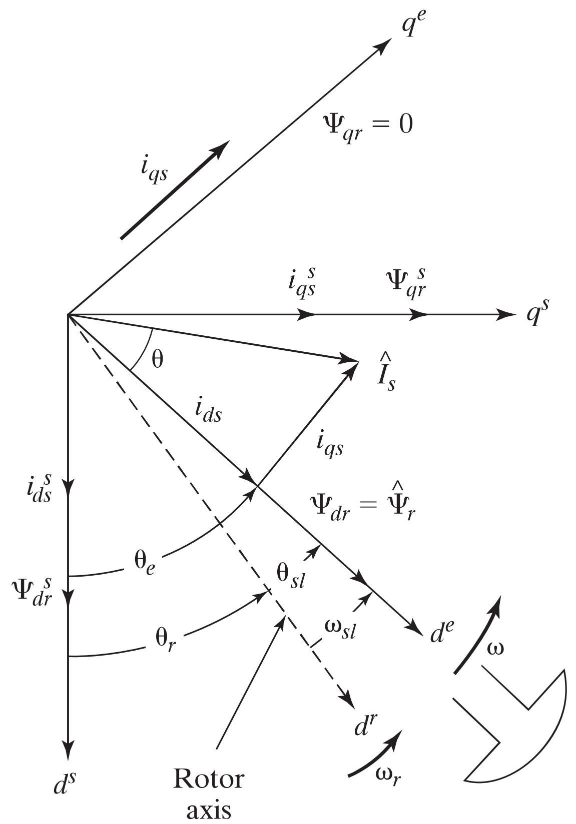


FIGURE 15.29

Phasor diagram showing space vector components for indirect vector control.

i_d currents to their reference values. The compensated i_q and i_d errors are then inverse transformed into the stator *a-b-c* reference frame for obtaining switching signals for the inverter via PWM or hysteresis comparators.

The various components of the space vectors are shown in Figure 15.29. The d^s–q^s axes are fixed on the stator, but the d^r–q^r axes, which are fixed on the rotor, are moving at speed ω_r. Synchronously rotating axes d^e–q^e are rotating ahead of the d^r–q^r axes by the positive slip angle θ_{sl} corresponding to slip frequency ω_{sl}. Because the rotor pole is directed on the d^e-axis and $\omega = \omega_r + \omega_{sl}$, we can write

$$\theta_s = \int \omega dt = \int (\omega_r + \omega_{sl}) dt = \theta_r + \theta_{sl} \tag{15.152}$$

The rotor position θ_s is not absolute, but is slipping with respect to the rotor at frequency ω_{sl}. For decoupling control, the stator flux component of current i_{ds} should be aligned on the d^e-axis, and the torque component of current i_{qs} should be on the q^e-axis.

This method uses a feed-forward scheme to generate ω_{sl}^* from i_{ds}^*, i_{qs}^*, and T_r. The rotor time constant T_r may not remain constant for all conditions of operation. Thus, with operating conditions, the slip speed ω_{sl}, which directly affects the developed torque and the rotor flux vector position, may vary widely. The indirect method requires the controller to be matched with the motor being driven. This is because the controller needs also to know some rotor parameter or parameters, which may vary according to the conditions of operation continuously. Many rotor time constant identification schemes may be adopted to overcome this problem.

Example 15.10 Finding the Rotor Flux Linkages

The parameters of an induction motor with an indirect vector controller are 6 hp, Y-connected, three phase, 60 Hz, four poles, 220 V, $R_s = 0.28\Omega$, $R_r = 0.17\Omega$, $L_m = 61\,\text{mH}$, $L_r = 56\,\text{mH}$, $L_s = 53\,\text{mH}$, $J = 0.01667\,\text{kg-m}^2$, rated speed $= 1800\,\text{rpm}$. Find (a) the rated rotor flux linkages and the corresponding stator currents i_{ds} and i_{qs}, (b) the total stator current I_s, (c) the torque angle θ_T, and (d) the slip gain K_{sl}.

Solution

$HP = 6\,\text{hp}$, $P_o = 745.7 \times 6 = 4474\,\text{W}$, $V_L = 220\,\text{V}$, $f = 60\,\text{Hz}$, $p = 4$, $R_s = 0.28\Omega$, $R_r = 0.17\Omega$, $L_m = 61\,\text{mH}$, $L_r = 56\,\text{mH}$, $L_s = 53\,\text{mH}$, $J = 0.01667\,\text{kg-m}^2$, $N = 1800\,\text{rpm}$. $\omega = 2\pi f = 2\pi \times 60 = 376.991\,\text{rad/s}$, $\omega_m = (2\pi N)/60 = (2\pi \times 1800)/60 = 188.496\,\text{rad/s}$, $\omega_r = (p/2) \times \omega_m = (4/2) \times 157.08 = 376.991\,\text{rad/s}$, $\omega_{c(\max)} = \omega_r = 314.159\,\text{rad/s}$, $V_{qs} = (\sqrt{2}V_L)/\sqrt{3} = (\sqrt{2} \times 220)/\sqrt{3} = 179.629\,rmV$, $V_{ds} = 0$

a. For $\omega_{sl} = 0$, Eq. (15.122) gives

$$\begin{pmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{pmatrix} = \begin{pmatrix} R_s & \omega L_s & 0 & \omega L_m \\ -\omega L_s & R_s & -\omega L_m & 0 \\ 0 & \omega_{sl} L_m & R_r & \omega_{sl} L_r \\ -\omega_{sl} L_m & 0 & -\omega_{sl} L_r & R_r \end{pmatrix}^{-1} \begin{pmatrix} v_{qs} \\ v_{ds} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.126 \\ 8.988 \\ 0 \\ 0 \end{pmatrix}$$

Thus, $i_{qs} = 0.126\,\text{A}$, $i_{ds} = 8.988\,\text{A}$, $i_{qr} = 0$, and $i_{dr} = 0$

From Eqs. (15.123) to (15.125), we get the stator flux linkages as

$$\Psi_{qs} = L_s i_{qs} + L_m i_{qr} = 53 \times 10^{-3} \times 0.126 + 61 \times 10^{-3} \times 0 = 6.678\,\text{mWb-turn}$$

$$\Psi_{ds} = L_s i_{ds} + L_m i_{dr} = 53 \times 10^{-3} \times 8.988 + 61 \times 10^{-3} \times 0 = 0.476\,\text{Wb-turn}$$

$$\Psi_s = \sqrt{\Psi_{qs}^2 + \Psi_{ds}^2} = \sqrt{(6.678 \times 10^{-2})^2 + 0.476^2} = 0.476\,\text{Wb-turn}$$

From Eqs. (15.126) to (15.128), we get the rotor flux linkages as

$$\Psi_{qr} = L_r i_{qr} + L_m i_{qs} = 56 \times 10^{-3} \times 0 + 61 \times 10^{-3} \times 0.126 = 6.678\,\text{mWb-turn}$$

$$\Psi_{dr} = L_r i_{dr} + L_m i_{ds} = 56 \times 10^{-3} \times 0 + 61 \times 10^{-3} \times 8.988 = 0.548\,\text{Wb-turn}$$

$$\Psi_r = \sqrt{\Psi_{qr}^2 + \Psi_{dr}^2} = \sqrt{(7.686 \times 10^{-2})^2 + 0.548^2} = 0.548\,\text{Wb-turn}$$

From Eqs. (15.129) to (15.131), we get the magnetizing flux linkages as

$$\Psi_{mq} = L_m(i_{qs} + i_{qr}) = 61 \times 10^{-3} \times (0.216 + 0) = 7.686\,\text{mWb}$$

$$\Psi_{md} = L_m + (i_{ds} + i_{dr}) = 61 \times 10^{-3} \times (8.988 + 0) = 0.548\,\text{Wb}$$

$$\Psi_m = \sqrt{\Psi_{mq}^2 + \Psi_{md}^2} = \sqrt{(7.686 \times 10^{-2})^2 + 0.548^2} = 0.548\,\text{Wb}$$

b. The flux-producing stator (or field) current to produce the mmf Ψ_m is

$$I_f = I_s\sqrt{i_{ds}^2 + i_{qs}^2} = \sqrt{8.988^2 + 0.216^2} = 8.989\,\text{A}$$

And Eq. (15.132) gives the corresponding torque as

$$T_d = \frac{3p}{4}(\Psi_{ds}i_{qs} - \Psi_{ds}i_{ds}) = \frac{3 \times 4}{4}(0.474 \times 0.216 - 6.678 \times 10^{-3} \times 8.988)$$
$$= 0\,(\text{as expected})$$

Equation (15.149) also gives the approximate value of the corresponding torque as

$$T_d = \frac{3p}{4} \times \frac{L_m^2}{L_r} i_{ds} i_{qs} = \frac{3 \times 4}{4} \times \frac{(61 \times 10^{-3})^2}{56 \times 10^{-3}} \times 8.988 \times 0.216 = 0.226\,\text{N.m}$$

c. The torque needed to produce the output power P_o is $T_e = P_o/\omega_m = 4744/188.496 = 23.736$ N.m. Equation (15.147) gives the torque constant $K_e = (3p/4)(L_m/L_r) = (3 \times 4/4) \times (61/56) = 3.268$ and the rotor current needed to produce the torque T_e is $I_r = i_{qs} = T_e/(K_e\Psi_r) = 23.736/(3.268 \times 0.548) = 13.247$ A.

Therefore, the total stator current is

$$I_s = \sqrt{I_f^2 + I_r^2} = \sqrt{8.989^2 + 13.247^2} = 16.01\text{A}$$

And the torque angle $\theta_T = \tan^{-1}(I_s/I_f) = \tan^{-1}(16.01/8.989) = 60.69°$

d. From Eq. (15.151),

$$\omega_{sl} = \frac{R_r}{L_r} \times \frac{I_s}{I_f} = \frac{0.17}{56 \times 10^{-3}} \times \frac{16.01}{8.989} = 5.06\,\text{rad/s}$$

$$K_{sl} = \frac{L_m R_r}{\psi_r L_r} = \frac{61 \times 10^{-3}}{0.548} \times \frac{0.17}{56 \times 10^{-3}} = 0.338$$

15.5.4 Direct Vector Control

The air-gap flux linkages in the stator *d*- and *q*-axes are used for respective leakage fluxes to determine the rotor flux linkages in the stator reference frame. The air-gap flux linkages are measured by installing quadrature flux sensors in the air gap, as shown in Figure 15.30.

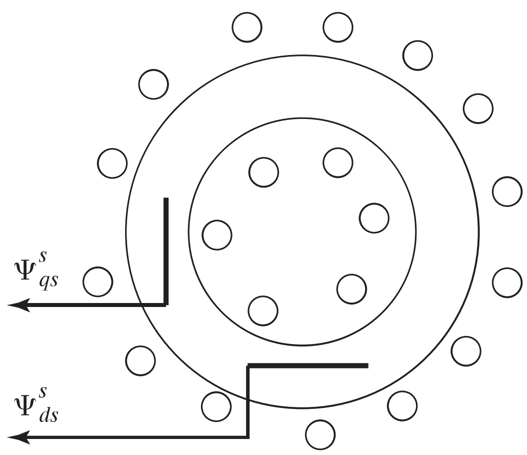


FIGURE 15.30

Quadrature sensors for air-gap flux for direct vector control.

Equations (15.123) to (15.131) in the stator reference frame can be simplified to get the flux linkages as given by

$$\Psi^s_{qr} = \frac{L_r}{L_m}\Psi^s_{qm} - L_{lr}i^s_{qs} \tag{15.153}$$

$$\Psi^s_{dr} = \frac{L_r}{L_m}\Psi^s_{dm} - L_{lr}i^s_{ds} \tag{15.154}$$

where the superscript^s stands for the stator reference frame. Figure 15.31 shows the phasor diagram for space-vector components. The current i_{ds} must be aligned in the direction of flux $\hat{\Psi}_r$ and i_{qs} must be perpendicular to $\hat{\Psi}_r$. The d^e–q^e frame is rotating at synchronous speed ω with respect to stationary frame d^s–q^s, and at any instant, the angular position of the d^e-axis with respect to the d^s-axis is θ_s in Eq. (15.122).

The stationary frame rotor flux vectors are given by

$$\Psi^s_{qr} = \hat{\Psi}_r \sin\theta_s \tag{15.155}$$

$$\Psi^s_{dr} = \hat{\Psi}_r \cos\theta_s \tag{15.156}$$

which give the unit vector components as

$$\cos\theta_s = \frac{\Psi^s_{dr}}{\hat{\Psi}_r} \tag{15.157}$$

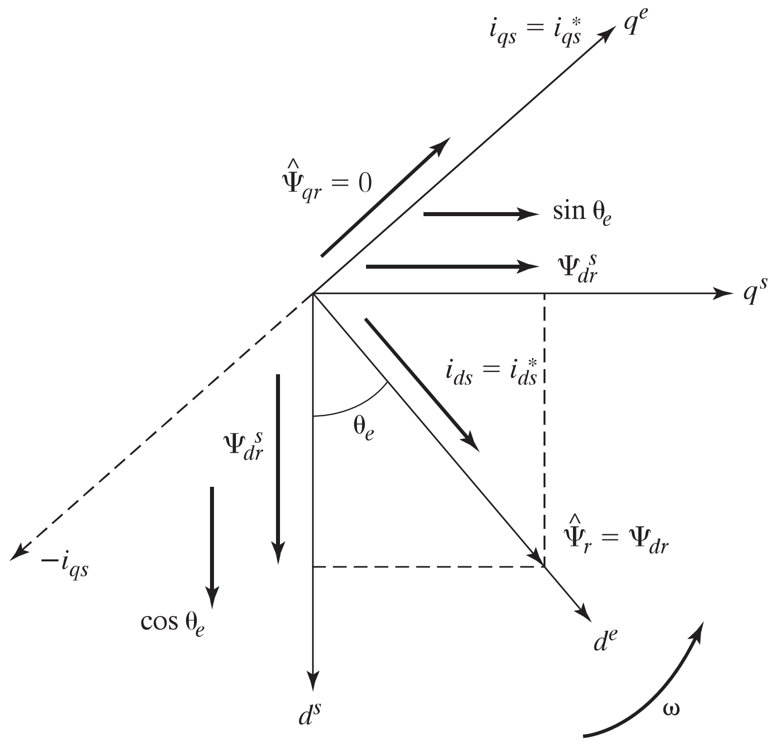


FIGURE 15.31

Phasors d^e–q^e and d^s–q^s for direct vector control.

$$\sin \theta_s = \frac{\Psi_{qr}^s}{\hat{\Psi}_r} \tag{15.158}$$

where

$$|\hat{\Psi}_r| = \sqrt{(\Psi_{dr}^2)^2 + (\Psi_{qr}^s)^2} \tag{15.159}$$

The flux signals Ψ_{dr}^s and Ψ_{qr}^s are generated from the machine terminal voltages and currents by using a voltage model estimator. The motor torque is controlled via the current i_{qs} and the rotor flux via the current i_{ds}. This method of control offers better low-speed performance than the IFOC. However, at high speed, the air-gap flux sensors reduce the reliability. The IFOC is normally preferred in practical applications. However, the d- and q-axes stator flux linkages of the motor may be computed from integrating the stator input voltages.

Key Points of Section 15.5

- The vector control technique uses the dynamic equivalent circuit of the induction motor. It decouples the stator current into two components: one providing the air-gap flux and the other producing the torque. It provides independent control of flux and torque, and the control characteristic is linearized.
- The stator currents are converted to a fictitious synchronously rotating reference frame aligned with the flux vector and are transformed back to the stator frame before feeding back to the machine. The indirect method is normally preferred over the direct method.

15.6 SYNCHRONOUS MOTOR DRIVES

Synchronous motors have a polyphase winding on the stator, also known as armature, and a field winding carrying a dc current on the rotor. There are two mmfs involved: one due to the field current and the other due to the armature current. The resultant mmf produces the torque. The armature is identical to the stator of induction motors, but there is no induction in the rotor. A synchronous motor is a constant-speed machine and always rotates with zero slip at the synchronous speed, which depends on the frequency and the number of poles, as given by Eq. (15.1). A synchronous motor can be operated as a motor or generator. The PF can be controlled by varying the field current. With cycloconverters and inverters the applications of synchronous motors in variable-speed drives are widening. The synchronous motors can be classified into six types:

1. Cylindrical rotor motors
2. Salient-pole motors
3. Reluctance motors
4. Permanent-magnet motors
5. Switched reluctance motors
6. Brushless dc and ac motors.

15.6.1 Cylindrical Rotor Motors

The field winding is wound on the rotor, which is cylindrical, and these motors have a uniform air gap. The reactances are independent on the rotor position. The equivalent circuit per phase, neglecting the no-load loss, is shown in Figure 15.32a, where R_a is the armature resistance per phase and X_s is the *synchronous reactance* per phase. V_f, which is dependent on the field current, is known as *excitation*, or *field* voltage.

The PF depends on the field current. The V-curves, which show the typical variations of the armature current against the excitation current, are shown in Figure 15.33. For the same armature current, the PF could be lagging or leading, depending on the excitation current I_f.

If θ_m is the lagging PF angle of the motor, Figure 15.32a gives

$$\overline{\mathbf{V}}_f = V_a \angle 0 - \bar{I}_a(R_a + jX_s) \tag{15.160}$$

$$= V_a \angle 0 - I_a(\cos\theta_m - j\sin\theta_m)(R_a + jX_s)$$

$$= V_a - I_aX_s\sin\theta_m - I_aR_a\cos\theta_m - jI_a(X_s\cos\theta_m - R_a\sin\theta_m) \tag{15.161a}$$

$$= V_f \angle \delta \tag{15.161b}$$

where

$$\delta = \tan^{-1}\frac{-(I_aX_s\cos\theta_m - I_aR_a\sin\theta_m)}{V_a - I_aX_s\sin\theta_m - I_aR_a\cos\theta_m} \tag{15.162}$$

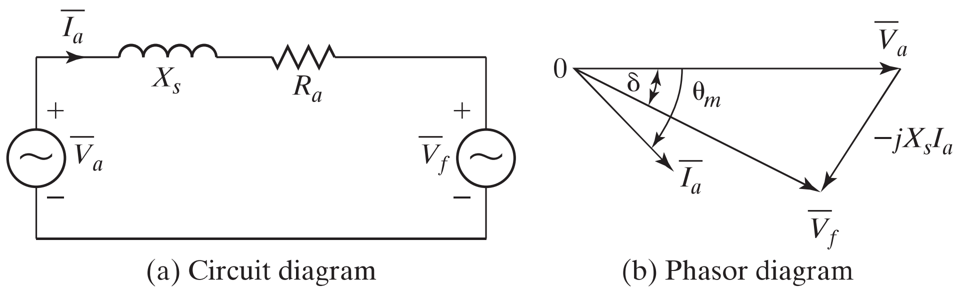
and

$$V_f = [(V_a - I_aX_s\sin\theta_m - I_aR_a\cos\theta_m)^2 + (I_aX_s\cos\theta_m - I_aR_a\sin\theta_m)^2]^{1/2} \tag{15.163}$$

The phasor diagram in Figure 15.32b yields

$$\overline{\mathbf{V}}_f = V_f(\cos\delta + j\sin\delta) \tag{15.164}$$

$$\bar{\mathbf{I}}_a = \frac{\overline{V}_a - \overline{V}_f}{R_a + jX_s} = \frac{[V_a - V_f(\cos\delta + j\sin\delta)](R_a - jX_s)}{R_a^2 + X_s^2} \tag{15.165}$$



(a) Circuit diagram (b) Phasor diagram

FIGURE 15.32

Equivalent circuit of synchronous motors.

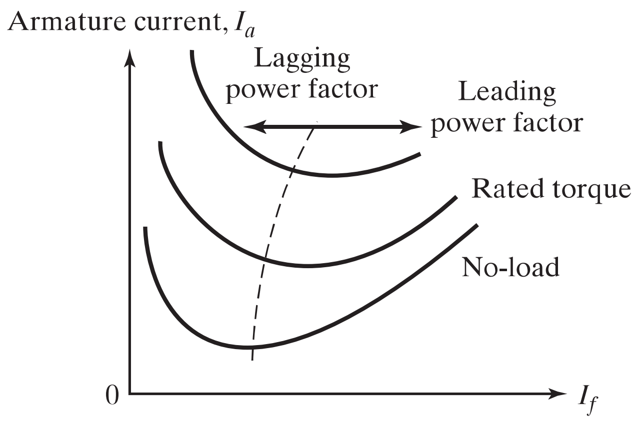


FIGURE 15.33
Typical V-curves of synchronous motors.

The real part of Eq. (15.165) becomes

$$I_a \cos \theta_m = \frac{R_a(V_a - V_f \cos \delta) - V_f X_s \sin \delta}{R_a^2 + X_s^2} \tag{15.166}$$

The input power can be determined from Eq. (15.166),

$$\begin{aligned} P_i &= 3 V_a I_a \cos \theta_m \\ &= \frac{3[R_a(V_a^2 - V_a V_f \cos \delta) - V_a V_f X_s \sin \delta]}{R_a^2 + X_s^2} \end{aligned} \tag{15.167}$$

The stator (or armature) copper loss is

$$P_{su} = 3 I_a^2 R_a \tag{15.168}$$

The gap power, which is the same as the developed power, is

$$P_d = P_g = P_i - P_{su} \tag{15.169}$$

If ω is the synchronous speed, which is the same as the rotor speed, the developed torque becomes

$$T_d = \frac{P_d}{\omega_s} \tag{15.170}$$

If the armature resistance is negligible, T_d in Eq. (15.170) becomes

$$T_d = -\frac{3 V_a V_f \sin \delta}{X_s \omega_s} \tag{15.171}$$

and Eq. (15.162) becomes

$$\delta = -\tan^{-1} \frac{I_a X_s \cos \theta_m}{V_a - I_a X_s \sin \theta_m} \tag{15.172}$$

For motoring δ is negative and torque in Eq. (15.171) becomes positive. In the case of generating, δ is positive and the power (and torque) becomes negative. The angle δ is called the *torque angle*. For a fixed voltage and frequency, the torque depends

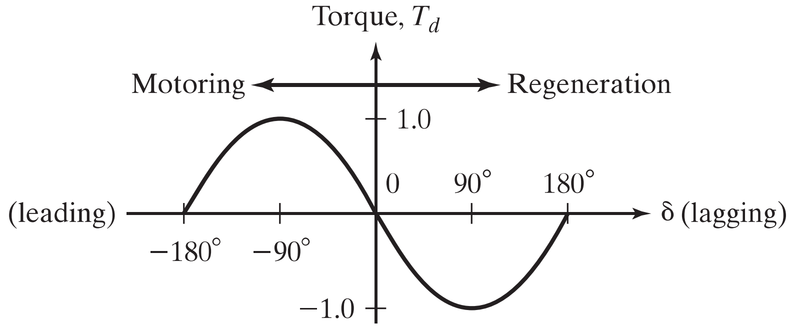


FIGURE 15.34
Torque versus torque angle with cylindrical rotor.

on the angle δ and is proportional to the excitation voltage V_f. For fixed values of V_f and δ, the torque depends on the voltage-to-frequency ratio and a constant volts/hertz control can provide speed control at a constant torque. If V_a, V_f, and δ remain fixed, the torque decreases with the speed and the motor operates in the field weakening mode.

If $\delta = 90°$, the torque becomes maximum and the maximum developed torque, which is called the *pull-out torque*, becomes

$$T_p = T_m = -\frac{3V_aV_f}{X_s\omega_s} \tag{15.173}$$

The plot of developed torque against the angle δ is shown in Figure 15.34. For stability considerations, the motor is operated in the positive slope of T_d–δ characteristics and this limits the range of torque angle, $-90° \leq \delta \leq 90°$.

Note: $\omega_s = \omega_m$ and $\omega_r = (p/2)\,\omega_m$

Example 15.11 Finding the Performance Parameters of a Cylindrical Rotor Synchronous Motor

A three-phase, 460-V, 60-Hz, six-pole, Y-connected cylindrical rotor synchronous motor has a synchronous reactance of $X_s = 2.5\ \Omega$ and the armature resistance is negligible. The load torque, which is proportional to the speed squared, is $T_L = 398\ \text{N}\cdot\text{m}$ at 1200 rpm. The PF is maintained at unity by field control and the voltage-to-frequency ratio is kept constant at the rated value. If the inverter frequency is 36 Hz and the motor speed is 720 rpm, calculate (a) the input voltage V_a, (b) the armature current I_a, (c) the excitation voltage V_f, (d) the torque angle δ, and (e) the pull-out torque T_p.

Solution

$\text{PF} = \cos\theta_m = 1.0$, $\theta_m = 0$, $V_{a(\text{rated})} = V_b = V_s = 460/\sqrt{3} = 265.58$ V, $p = 6$, $\omega = 2\pi \times 60 = 377$ rad/s, $\omega_b = \omega_s = \omega_m = 2 \times 377/6 = 125.67$ rad/s or 1200 rpm, and $d = V_b/\omega_b = 265.58/125.67 = 2.1133$. At 720 rpm,

$$T_L = 398 \times \left(\frac{720}{1200}\right)^2 = 143.28\ \text{N}\cdot\text{m} \quad \omega_s = \omega_m = 720 \times \frac{\pi}{30} = 75.4\ \text{rad/s}$$

$$P_0 = 143.28 \times 75.4 = 10{,}803\ \text{W}$$

a. $V_a = d\omega_s = 2.1133 \times 75.4 = 159.34$ V.

b. $P_0 = 3V_aI_a\text{PF} = 10{,}803$ or $I_a = 10{,}803/(3 \times 159.34) = 22.6$ A.

c. From Eq. (15.160),

$$\overline{\mathbf{V}}_f = 159.34 - 22.6 \times (1 + j0)(j2.5) = 169.1 \angle -19.52^\circ$$

d. The torque angle, $\delta = -19.52°$.

e. From Eq. (15.173),

$$T_p = \frac{3 \times 159.34 \times 169.1}{2.5 \times 75.4} = 428.82 \text{ N} \cdot \text{m}$$

15.6.2 Salient-Pole Motors

The armature of salient-pole motors is similar to that of cylindrical rotor motors. However, due to saliency, the air gap is not uniform and the flux is dependent on the position of the rotor. The field winding is normally wound on the pole pieces. The armature current and the reactances can be resolved into direct- and quadrature-axis components. I_d and I_q are the components of the armature current in the direct (or d) axis and quadrature (or q) axis, respectively. X_d and X_q are the d-axis reactance and q-axis reactance, respectively. Using Eq. (15.160), the excitation voltage becomes

$$\overline{\mathbf{V}}_f = \overline{\mathbf{V}}_a - jX_d\overline{\mathbf{I}}_d - jX_q\overline{\mathbf{I}}_q - R_a\overline{\mathbf{I}}_a$$

For negligible armature resistance, the phasor diagram is shown in Figure 15.35. From the phasor diagram,

$$I_d = I_a \sin(\theta_m - \delta) \tag{15.174}$$

$$I_q = I_a \cos(\theta_m - \delta) \tag{15.175}$$

$$I_d X_d = V_a \cos\delta - V_f \tag{15.176}$$

$$I_q X_q = V_a \sin\delta \tag{15.177}$$

Substituting I_q from Eq. (15.175) in Eq. (15.177), we have

$$\begin{aligned} V_a \sin\delta &= X_q I_a \cos(\theta_m - \delta) \\ &= X_q I_a (\cos\delta \cos\theta_m + \sin\delta \sin\theta_m) \end{aligned} \tag{15.178}$$

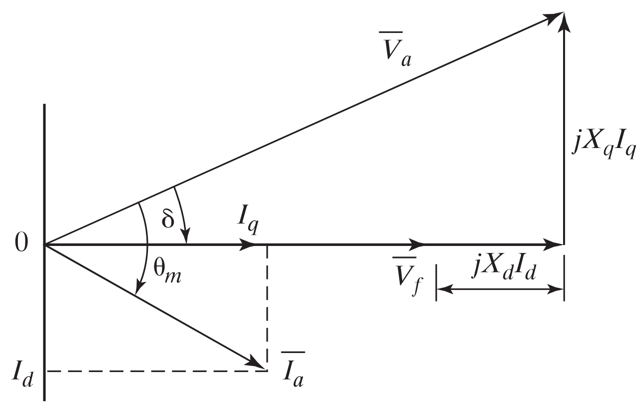


FIGURE 15.35

Phase diagram for salient-pole synchronous motors.

Dividing both sides by cos δ and solving for δ gives

$$\delta = -\tan^{-1}\frac{I_a X_q \cos\theta_m}{V_a - I_a X_q \sin\theta_m} \tag{15.179}$$

where the negative sign signifies that V_f lags V_a. If the terminal voltage is resolved into a d-axis and a q-axis,

$$V_{ad} = -V_a \sin\delta \text{ and } V_{aq} = V_a \cos\delta$$

The input power becomes

$$\begin{aligned} P &= -3(I_d V_{ad} + I_q V_{aq}) \\ &= 3I_d V_a \sin\delta - 3I_q V_a \cos\delta \end{aligned} \tag{15.180}$$

Substituting I_d from Eq. (15.176) and I_q from Eq. (15.177) in Eq. (15.180) yields

$$P_d = -\frac{3V_a V_f}{X_d}\sin\delta - \frac{3V_a^2}{2}\left[\frac{X_d - X_q}{X_d X_q}\sin 2\delta\right] \tag{15.181}$$

Dividing Eq. (15.181) by speed gives the developed torque as

$$T_d = -\frac{3V_a V_f}{X_d \omega_s}\sin\delta - \frac{3V_a^2}{2\omega_s}\left[\frac{X_d - X_q}{X_d X_q}\sin 2\delta\right] \tag{15.182}$$

The torque in Eq. (15.182) has two components. The first component is the same as that of the cylindrical rotor if X_d is replaced by X_s and the second component is due to the rotor saliency. The typical plot of T_d against torque angle is shown in Figure 15.36, where the torque has a maximum value at $\delta = \pm\delta_m$. For stability the torque angle is limited in the range of $-\delta_m \le \delta \le \delta_m$ and in this stable range, the slope of the T_d–δ characteristic is higher than that of cylindrical rotor motor.

15.6.3 Reluctance Motors

The reluctance motors are similar to the salient-pole motors, except that there is no field winding on the rotor. The armature circuit, which produces rotating magnetic field in the air gap, induces a field in the rotor that has a tendency to align with the

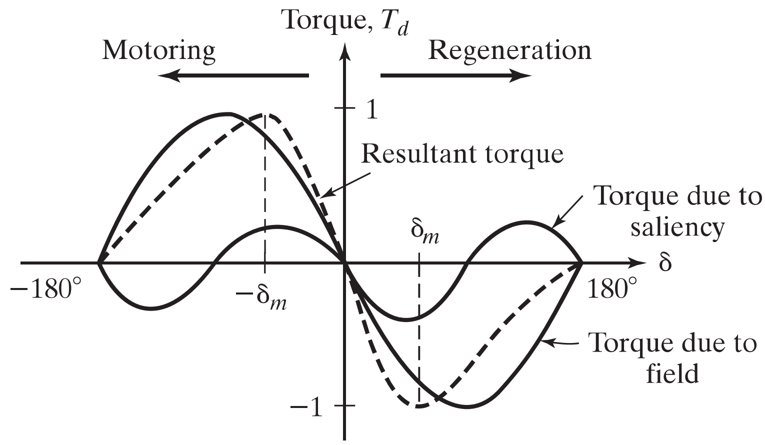


FIGURE 15.36

Torque versus torque angle with salient-pole rotor.

armature field. The reluctance motors are very simple and are used in applications where a number of motors are required to rotate in synchronism. These motors have low-lagging PF, typically in the range 0.65 to 0.75.

With $V_f = 0$, Eq. (15.182) can be applied to determine the reluctance torque,

$$T_d = -\frac{3V_a^2}{2\omega_s}\left[\frac{X_d - X_q}{X_d X_q}\sin 2\delta\right] \tag{15.183}$$

where

$$\delta = -\tan^{-1}\frac{I_a X_q \cos\theta_m}{V_a - I_a X_q \sin\theta_m} \tag{15.184}$$

The pull-out torque for $\delta = -45°$ is

$$T_p = \frac{3V_a^2}{2\omega_s}\left[\frac{X_d - X_q}{X_d X_q}\right] \tag{15.185}$$

Example 15.12 Finding the Performance Parameters of a Reluctance Motor

A three-phase, 230-V, 60-Hz, four-pole, Y-connected reluctance motor has $X_d = 22.5\ \Omega$ and $X_q = 3.5\ \Omega$. The armature resistance is negligible. The load torque is $T_L = 12.5\ \text{N}\cdot\text{m}$. The voltage-to-frequency ratio is maintained constant at the rated value. If the supply frequency is 60 Hz, determine (a) the torque angle δ, (b) the line current I_a, and (c) the input PF.

Solution

$T_L = 12.5\ \text{N}\cdot\text{m}$, $V_{a(\text{rated})} = V_b = 230/\sqrt{3} = 132.79$ V, $p = 4$, $\omega = 2\pi \times 60 = 377$ rad/s, $\omega_b = \omega_s = \omega_m = 2 \times 377/4 = 188.5$ rad/s or 1800 rpm, and $V_a = 132.79$ V.

a. $\omega_s = 188.5$ rad/s. From Eq. (15.183),

$$\sin 2\delta = -\frac{12.5 \times 2 \times 188.5 \times 22.5 \times 3.5}{3 \times 132.79^2 \times (22.5 - 3.5)}$$

and $\delta = -10.84°$.

b. $P_0 = 12.5 \times 188.5 = 2356$ W. From Eq. (15.184),

$$\tan(10.84°) = \frac{3.5I_a\cos\theta_m}{132.79 - 3.5I_a\sin\theta_m}$$

and $P_0 = 2356 = 3 \times 132.79 I_a \cos\theta_m$. From these two equations, I_a and θ_m can be determined by an iterative method of solution, which yields $I_a = 9.2$ A and $\theta_m = 49.98°$.

c. PF $= \cos(49.98°) = 0.643$.

15.6.4 Switched Reluctance Motors

A switched reluctance motor (SRM) is a variable reluctance step motor. A cross-sectional view is shown in Figure 15.37a. Three phases $(q = 3)$ are shown with six stator teeth, $N_s = 6$, and four rotor teeth, $N_r = 4$. N_r is related to N_s and q

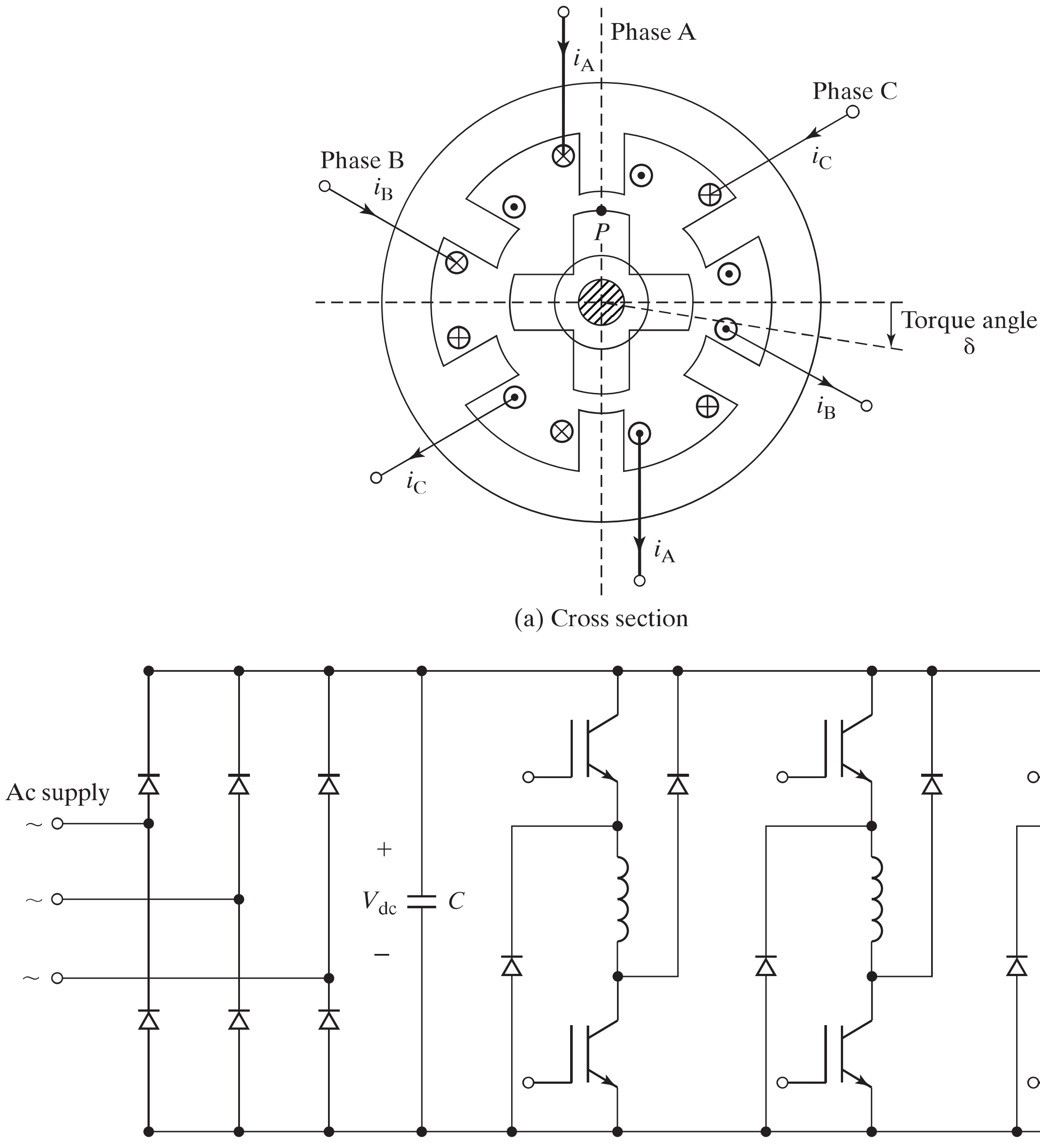


FIGURE 15.37
Switched reluctance motor.

by $N_r = N_s \pm N_s/q$. Each phase winding is placed on two diametrically opposite teeth. If phase A is excited by a current i_a, a torque is developed and it causes a pair of rotor poles to be magnetically aligned with the poles of phase A. If the subsequent phase B and phase C are excited in sequence, a further rotation can take place. The motor speed can be varied by exciting in sequence phases A, B, and C. A commonly used circuit to drive an SRM is shown in Figure 15.37b. An absolute position sensor is usually required to directly control the angles of stator excitation with respect to the rotor position. A position feedback control is provided in generating the gating signals. If the switching takes place at a fixed rotor position relative to the rotor poles, an SRM would exhibit the characteristics of a dc series motor. By varying the rotor position, a range of operating characteristics can be obtained [16, 17].

15.6.5 Permanent-Magnet Motors

The permanent-magnet motors are similar to salient-pole motors, except that there is no field winding on the rotor and the field is provided by mounting permanent magnets at the rotor. The excitation voltage cannot be varied. For the same frame size, permanent-magnet motors have higher pull-out torque. The equations for the salient-pole motors may be applied to permanent-magnet motors if the excitation voltage V_f is assumed constant. The elimination of field coil, dc supply, and slip rings reduces the motor loss and the complexity. These motors are also known as *brushless motors* and find increased applications in robots and machine tools. A permanent-magnet motor can be fed by either rectangular current or sinusoidal current. The rectangular current-fed motors, which have concentrated windings on the stator inducing a square or trapezoidal voltage, are normally used in low-power drives. The sinusoidal current-fed motors, which have distributed windings on the stator, provide smoother torque and are normally used in high-power drives.

Taking the rotor frame as the reference, the position of the rotor magnets determines the stator voltages and currents, the instantaneous induced emfs, and subsequently the stator currents and torque of the machine. The equivalent q- and d-axis stator windings are transformed to the reference frames that are revolving at rotor speed ω_r. Thus, there is zero speed differential between the rotor and stator magnetic fields, and the stator q- and d-axis windings have a fixed phase relationship with the rotor magnet axis, that is, the d-axis.

The stator flux linkage equations are [23]

$$v_{qs} = R_q i_{qs} + D\Psi_{qs} + \omega_r \Psi_{ds} \tag{15.186}$$

$$v_{ds} = R_d i_{ds} + D\Psi_{ds} - \omega_r \Psi_{qs} \tag{15.187}$$

where R_q and R_d are the quadrature- and direct-axis winding resistances, which are equal to stator resistance R_s. The q- and d-axis stator flux linkages in the rotor reference frames are

$$\Psi_{qs} = L_q i_{qs} + L_m i_{qr} \tag{15.188}$$

$$\Psi_{ds} = L_d i_{ds} + L_m i_{dr} \tag{15.189}$$

where L_m is the mutual inductance between the stator winding and rotor magnets.

L_q and L_d are the self-inductances of the stator q- and d-axis windings. These become equal to the stator inductance L_s only when the rotor magnets have an arc of electrical 180°. As the rotor magnets and the stator q- and d-axis windings are fixed in space, the winding inductances do not change in rotor reference frames. The rotor flux is along the d-axis, so the d-axis rotor current is i_{dr}. The q-axis current in the rotor is zero, that is, $i_{qr} = 0$, because there is no flux along this axis in the rotor. Equations (15.188) and (15.189) for the flux linkage can be written as

$$\Psi_{qs} = L_q i_{qs} \tag{15.190}$$

$$\Psi_{ds} = L_d i_{ds} + L_m i_{dr} \tag{15.191}$$

Substituting these flux linkages into the stator voltage Eqs. (15.186) and (15.187) gives the stator equations as

$$\begin{pmatrix} v_{qs} \\ v_{ds} \end{pmatrix} = \begin{pmatrix} R_q + L_q D & \omega_r L_d \\ -\omega_r L_q & R_d + L_d D \end{pmatrix} \begin{pmatrix} i_{qs} \\ i_{ds} \end{pmatrix} + \begin{pmatrix} \omega_r L_m i_{dr} \\ 0 \end{pmatrix} \tag{15.192}$$

The electromagnetic torque is given by

$$T_e = \frac{3}{2}\frac{p}{2}(\Psi_{ds} i_{qs} - \Psi_{qs} i_{ds}) \tag{15.193}$$

which, upon substitution of the flux linkages from Eqs. (15.190) and (15.191) in terms of the inductances and currents, gives

$$T_e = \frac{3}{2}\frac{p}{2}\left[L_m i_{dr} i_{qs} + (L_d - L_q) i_{qs} i_{ds}\right] \tag{15.194}$$

and the rotor flux linkage that links the stator is

$$\Psi_r = L_m i_{dr} \tag{15.195}$$

The rotor flux linkage can be considered constant except for temperature effects. Assume sinusoidal three-phase inputs as follows:

$$i_{as} = i_s \sin(\omega_r t + \delta) \tag{15.196}$$

$$i_{bs} = i_s \sin\left(\omega_r t + \delta - \frac{2\pi}{3}\right) \tag{15.197}$$

$$i_{cs} = i_s \sin\left(\omega_r t + \delta + \frac{2\pi}{3}\right) \tag{15.198}$$

where ω_r is the electrical rotor speed and δ is the angle between the rotor field and stator current phasor, known as the *torque angle*. The rotor field is traveling at a speed of ω_r rad/s.

Therefore, the q- and d-axis stator currents in the rotor reference frame for a balanced three-phase operation are given by

$$\begin{pmatrix} i_{qs} \\ i_{ds} \end{pmatrix} = \frac{2}{3}\begin{bmatrix} \cos \omega_r t & \cos\left(\omega_r t - \frac{2\pi}{3}\right) & \cos\left(\omega_r t + \frac{2\pi}{3}\right) \\ \sin \omega_r t & \sin\left(\omega_r t - \frac{2\pi}{3}\right) & \sin\left(\omega_r t + \frac{2\pi}{3}\right) \end{bmatrix} \begin{pmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{pmatrix} \tag{15.199}$$

Substituting the Eqs. from (15.196) to (15.198) into (15.199) gives the stator currents in the rotor reference frames [23]:

$$\begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} = i_s \begin{bmatrix} \sin \delta \\ \cos \delta \end{bmatrix} \tag{15.200}$$

The q- and d-axis currents are constants in rotor reference frames, since δ is a constant for a given load torque. These are very similar to armature and field currents in the separately excited dc machine. The q-axis current is equivalent to the armature current of the dc machine. The d-axis current is field current, but not in its entirety. It is only a partial field current; the other part is contributed by the equivalent current source representing the permanent magnet field.

Substituting Eq. (15.200) into the electromagnetic torque Eq. (15.194) gives the torque as

$$T_e = \frac{3}{2}\frac{p}{2}\left[\Psi_{rf} i_s \sin\delta + \frac{1}{2}(L_d - L_q) i_s^2 \sin 2\delta\right] \tag{15.201}$$

which for $\delta = \pi/2$ becomes

$$T_e = \frac{3}{2} \times \frac{p}{2} \times \Psi_{rf} i_s = K_f \Psi_{rf} i_s \text{ N} \cdot \text{m} \tag{15.202}$$

where $K_f = 3p/(4)$

Thus, the torque equation is similar to that of the torque generated in the dc motor and vector-controlled induction motor. If the torque angle δ is maintained at 90° and flux is kept constant, then the torque is controlled by the stator-current magnitude and the operation is very similar to that of the armature-controlled separately excited dc motor. The electromagnetic torque is positive for the motoring action, if δ is positive. The rotor flux linkage Ψ_r is positive. The phasor diagram for an arbitrary torque angle δ is shown in Figure 15.38.

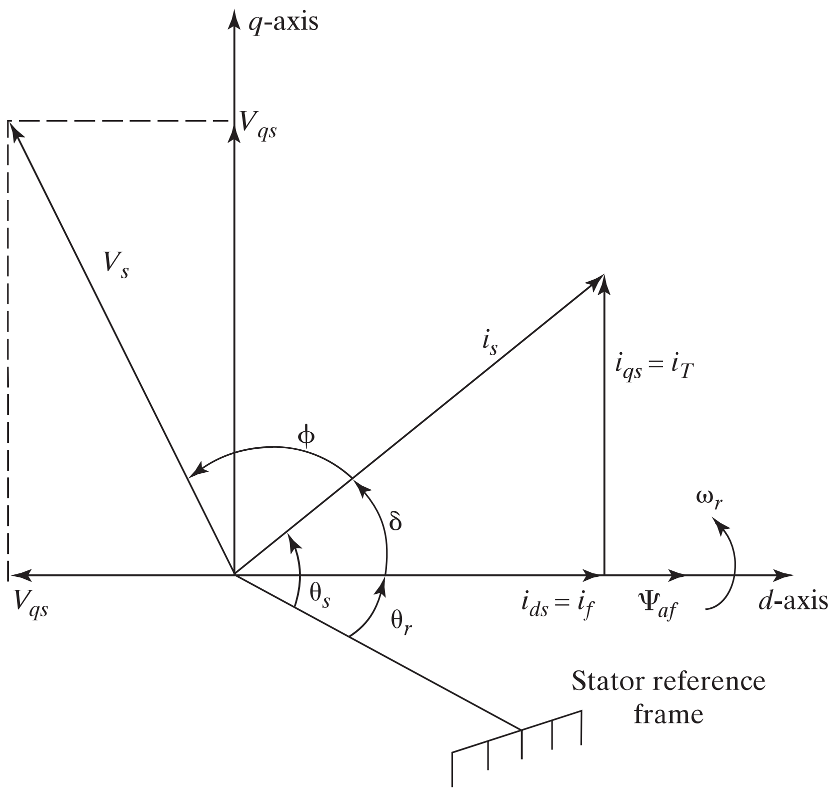


FIGURE 15.38

Phasor diagram of the PM synchronous machine.

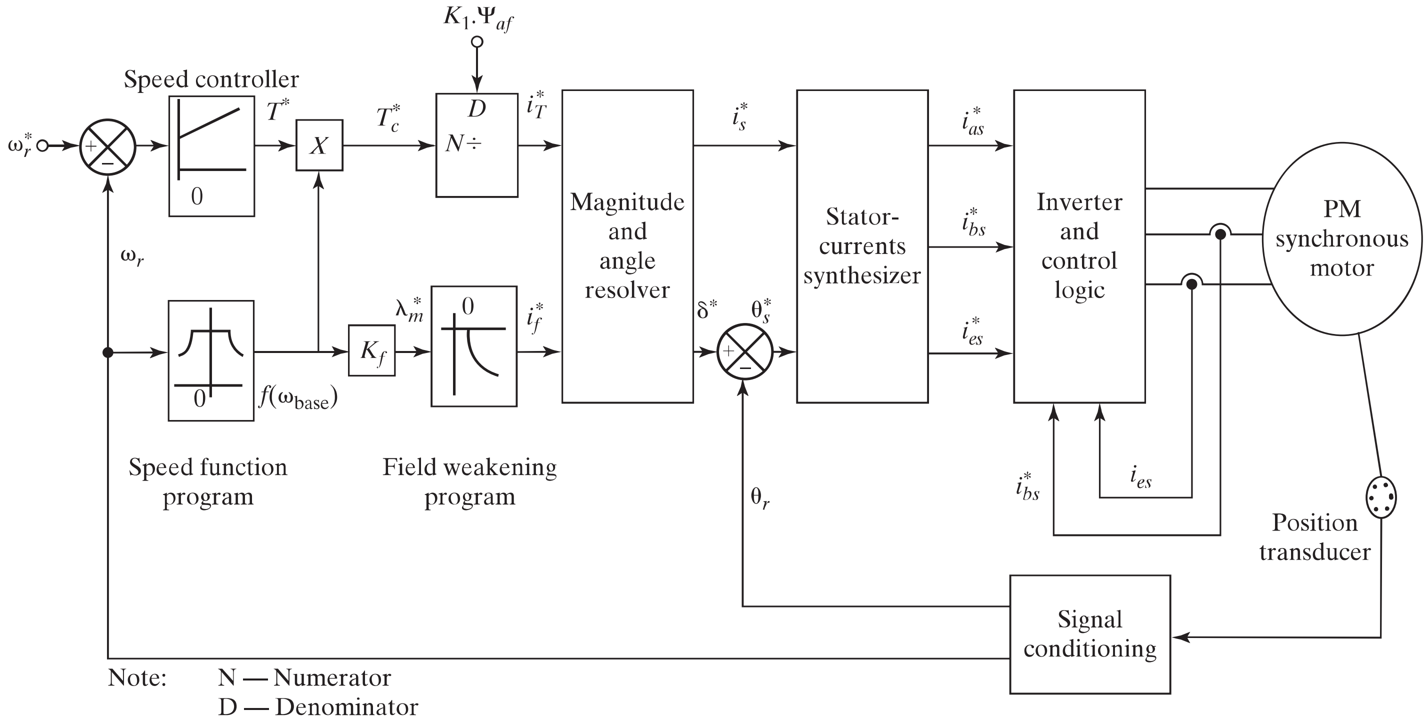


FIGURE 15.39

Block diagram of the vector-controlled PM synchronous motor drive [23].

It should be noted that i_{qs} is the torque-producing component of stator current and i_{ds} is the flux-producing component of stator current. The mutual flux linkage, which is the resultant of the rotor flux linkages and stator flux 1inkages, is given by

$$\Psi_m = \sqrt{(\Psi_{fr} + L_d i_{ds})^2 + (L_q i_{qs})^2} \quad (\text{Wb} - \text{turn}) \tag{15.203}$$

If δ is greater than $\pi/2$, i_{ds} becomes negative. Hence, the resultant mutual flux linkages decrease and cause the flux-weakening in the PM synchronous motor drives. If δ is negative with respect to the rotor or mutual flux linkage, the machine becomes a generator.

The schematic of the vector-controlled PM synchronous motor drive is shown in Figure 15.39 [23]. The torque reference is a function of the speed error, and the speed controller is usually of PI type. For fast response of the speed, a PID controller is often used. The product of torque reference and air-gap flux linkage Ψ_m^* generates the torque-producing component i_T^* of the stator current.

15.6.6 Closed-Loop Control of Synchronous Motors

The typical characteristics of torque, current, and excitation voltage against frequency ratio β are shown in Figure 15.40a. There are two regions of operation: constant torque and constant power. In the constant-torque region, the volts/hertz is maintained constant, and in the constant-power region, the torque decreases with frequency. Speed–torque characteristics for different frequencies are shown in Figure 15.40b. Similar to induction motors, the speed of synchronous motors can be controlled by varying the

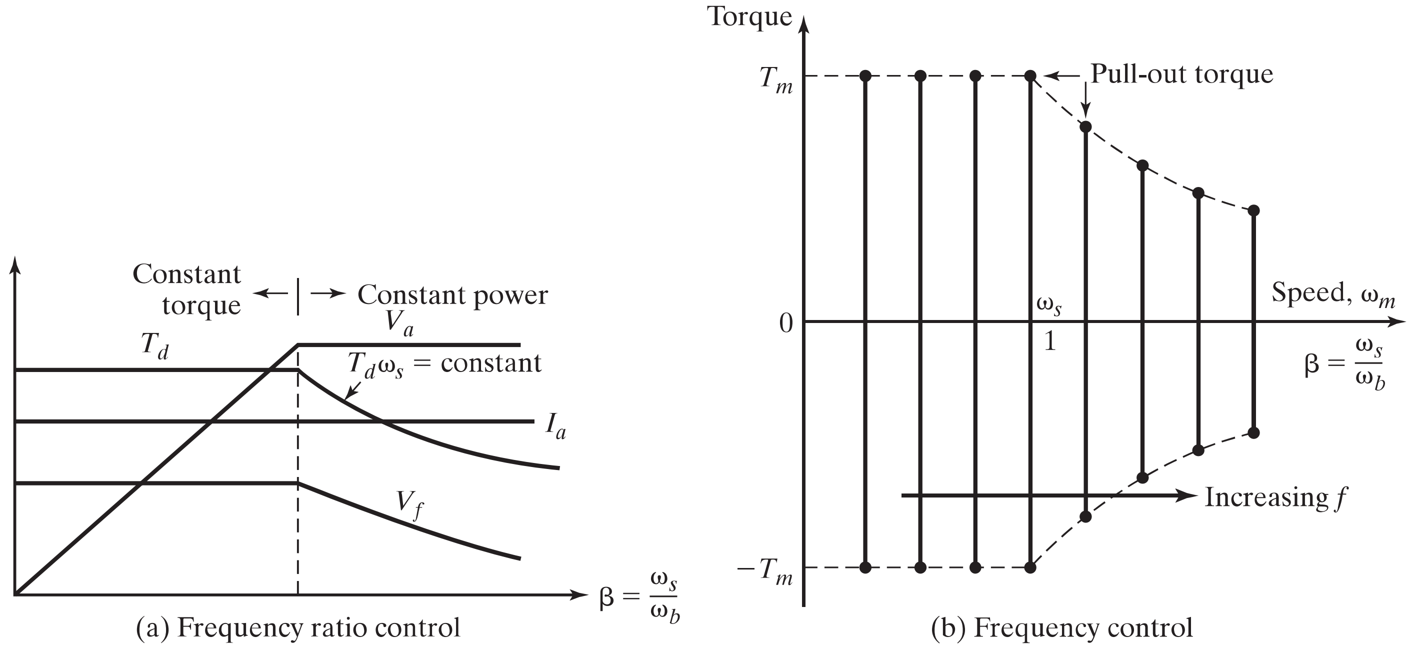


FIGURE 15.40

Torque–speed characteristics of synchronous motors.

voltage, frequency, and current. There are various configurations for closed-loop control of synchronous motors. A basic arrangement for constant volts/hertz control of synchronous motors is shown in Figure 15.41, where the speed error generates the frequency and voltage command for the PWM inverter. Because the speed of synchronous motors depends on the supply frequency only, they are employed in multimotor drives requiring accurate speed tracking between motors, as in fiber spinning mills, paper mills, textile mills, and machine tools.

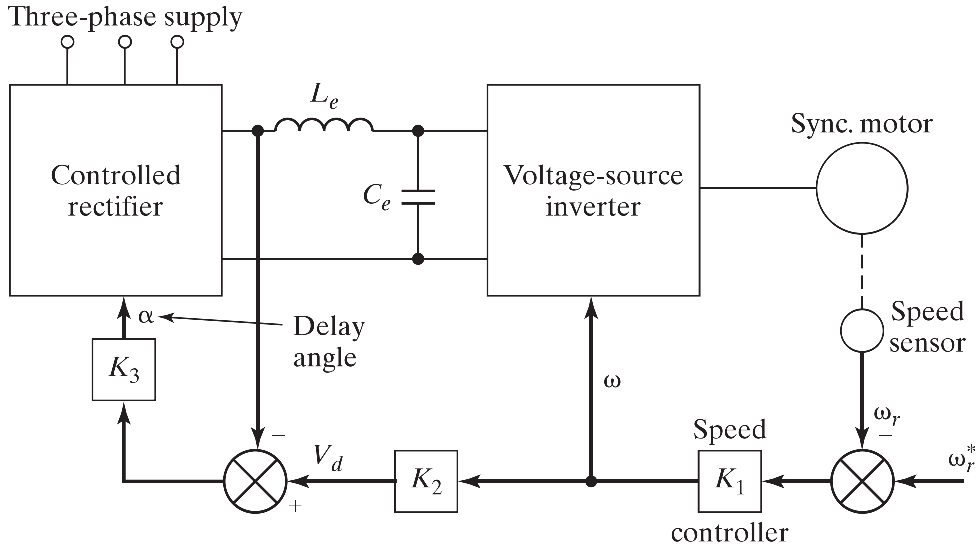


FIGURE 15.41

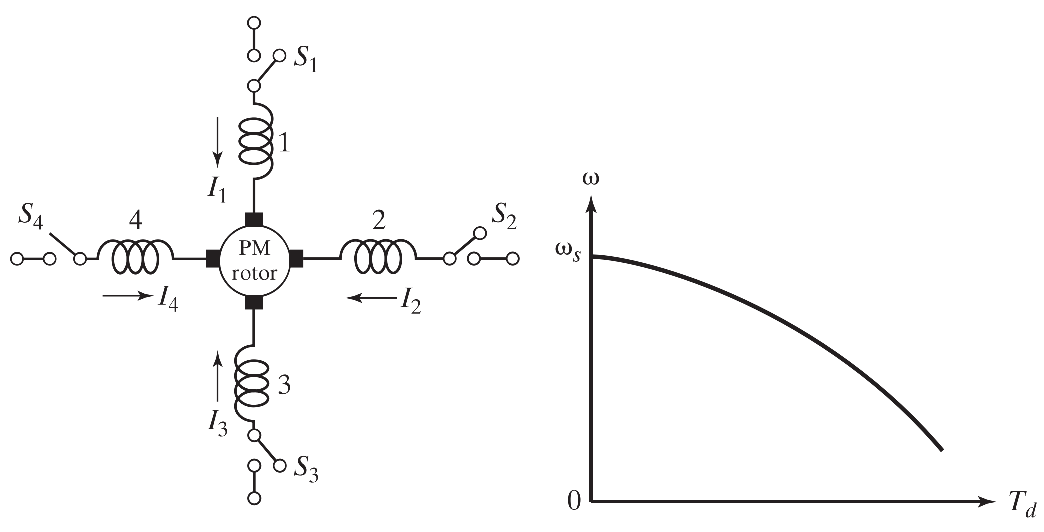
Volts/hertz control of synchronous motors.

15.6.7 Brushless Dc and Ac Motor Drives

A brushless dc motor consists of a multiphase winding wound on a nonsalient stator and a radically magnetized PM rotor [24]. Figure 15.42a shows the schematic diagram of a brushless dc motor. The multiphase winding may be a single coil or distributed over the pole span. Dc or ac voltage can be applied to individual phase windings through a sequential switching operation to achieve the necessary commutation to impart rotation. If winding 1 is energized, the PM rotor aligns with the magnetic field produced by winding 1. When winding 1 is switched off while winding 2 is turned on, the rotor is made to rotate to line up with the magnetic field of winding 2, and so on. The rotor position can be detected by using either Hall-effect or photoelectric devices. The desired speed–torque characteristic of a brushless dc motor as shown in Figure 15.42b can be obtained by the control of the magnitude and the rate of switching of the phase currents.

Brushless drives are basically synchronous motor drives in self-control mode. The armature supply frequency is changed in proportion to the rotor speed changes so that the armature field always moves at the same speed as the rotor. The self-control ensures that for all operating points the armature and rotor fields move exactly at the same speed. This prevents the motor from pulling out of step, hunting oscillations, and instability due to a step change in torque or frequency. The accurate tracking of the speed is normally realized with a rotor position sensor. The PF can be maintained at unity by varying the field current. The block diagrams of a self-controlled synchronous motor fed from a three-phase inverter or a cycloconverter are shown in Figure 15.43.

For an inverter-fed drive, as shown in Figure 15.43a, the input source is dc. Depending on the type of inverter, the dc source could be a current source, a constant current, or a controllable voltage source. The inverter's frequency is changed in proportion to the speed so that the armature and rotor mmf waves revolve at the same speed, thereby producing a steady torque at all speeds, as in a dc motor. The rotor position and



(a) Schematic (b) Speed–torque characteristic

FIGURE 15.42

Brushless dc motor.

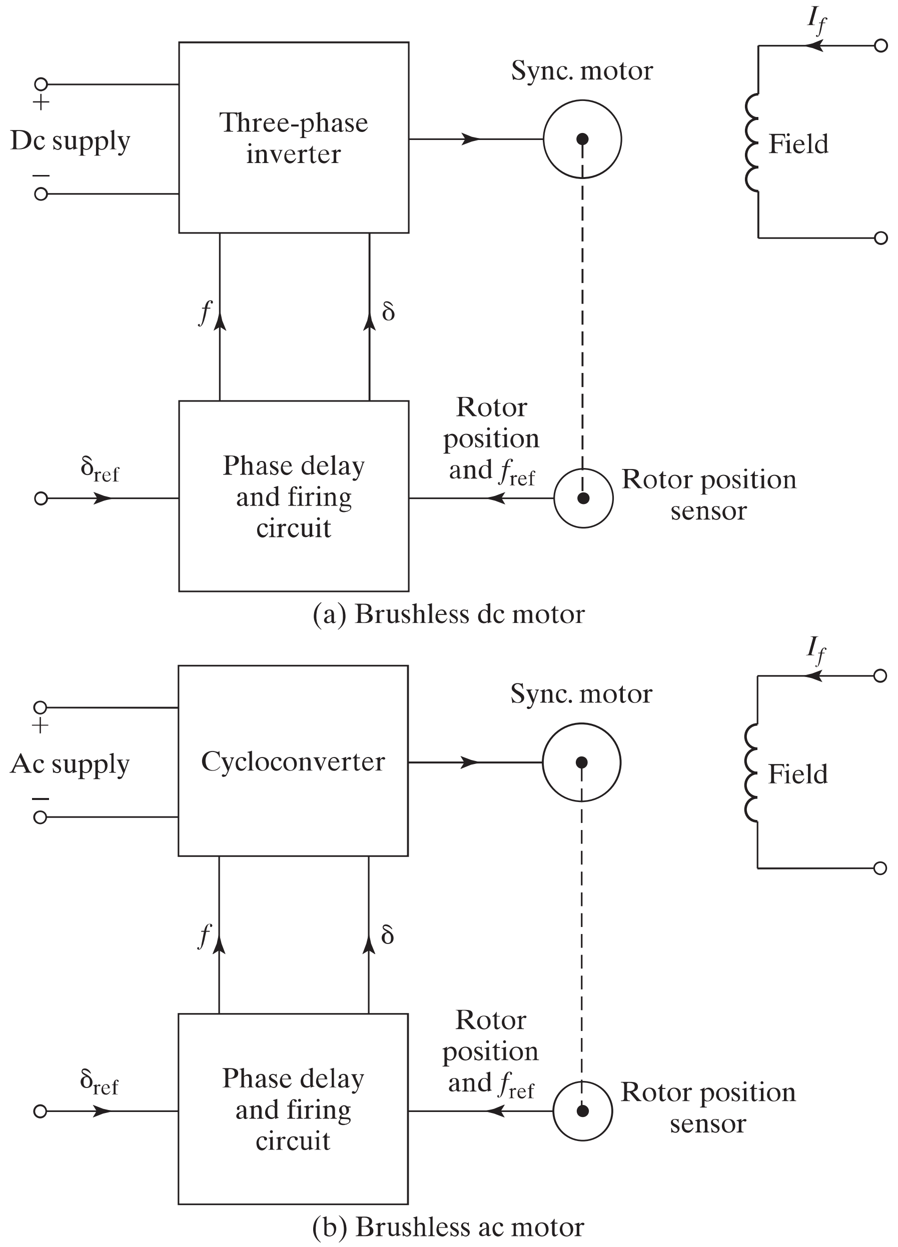


FIGURE 15.43

Self-controlled synchronous motors.

inverter perform the same function as the brushes and commutator in a dc motor. Due to the similarity in operation with a dc motor, an inverter-fed self-controlled synchronous motor is known as a *commutatorless dc motor*. If the synchronous motor is a permanent-magnet motor, a reluctance motor, or a wound field motor with a brushless excitation, it is known as a *brushless* and *commutatorless dc motor* or simply a *brushless dc motor*. Connecting the field in series with the dc supply gives the characteristics of a dc series motor. The brushless dc motors offer the characteristics of dc motors and do not have limitations such as frequent maintenance and inability to operate in explosive environments. They are finding increasing applications in servo drives [18].

If the synchronous motor is fed from an ac source as shown in Figure 15.43b, it is called a *brushless* and *commutatorless ac motor* or simply *a brushless ac motor*.

These ac motors are used for high-power applications (up to megawatt range), such as compressors, blowers, fans, conveyers, steel rolling mills, large ship steering, and cement plants. The self-controlled motor is alsoused for starting large synchronous motors in gas turbine and pump storage power plants.

Key Points of Section 15.7

- A synchronous motor is a constant-speed machine and always rotates with zero slip at the synchronous speed.
- The torque is produced by the armature current in the stator winding and the field current in the rotor winding.
- The cycloconverters and inverters are used for applications of synchronous motors in variable-speed drives.

15.7 DESIGN OF SPEED CONTROLLER FOR PMSM DRIVES

The design of the speed controller is important to obtain the desired transient and steady-state characteristics of the drive system. A proportional-plus-integral controller is sufficient for many industrial applications. The selection of the gain and time constants of the controller can be simplified if the d-axis stator current is assumed to be zero. Under the assumption, that is, $i_{ds} = 0$, the system becomes linear and resembles that of a separately excited dc motor with constant excitation.

15.7.1 System Block Diagram

Assuming $i_{ds} = 0$, Eq. (15.192) gives the motor q-axis voltage equation as

$$v_{qs} = (R_q + L_q D) i_{qs} + \omega_r L_m i_{dr} = (R_q + L_q D) i_{qs} + \Psi_r \omega_r \tag{15.204}$$

and the electromechanical equation is given by

$$\frac{P}{2}(T_e - T_L) = JD\omega_r + B_1\omega_r \tag{15.205}$$

where the electromagnetic torque T_e is given in Eq. (15.202)

$$T_e = \frac{3}{2} \times \frac{p}{2} \times \Psi_{rf} i_{qs} \tag{15.206}$$

and the load torque for assuming friction only is given by

$$T_L = B_L \omega_r \tag{15.207}$$

Substituting Eqs. (15.206) and (15.207) into Eq. (15.205) gives the electromechanical equation as

$$(JD + B_1)\omega_r + \left[\frac{3}{2} \times \left(\frac{p}{2}\right)^2 \Psi_r\right] i_{qs} = K_T i_{qs} \tag{15.208}$$

where

$$B_T = B_L + B_1 \tag{15.209}$$

$$K_T = \frac{3}{2} \times \left(\frac{p}{2}\right)^2 \Psi_r \tag{15.210}$$

The block diagram representing Eqs. (15.204) and (15.208) is shown in Figure 15.44 including the current- and speed-feedback loops, where B_t is the viscous friction of the motor and the load.

The inverter can be modeled as a gain with a time lag as given by

$$G_r(s) = \frac{K_{in}}{1 + sT_{in}} \tag{15.211}$$

where

$$K_{in} = \frac{0.65\, V_{\mathrm{dc}}}{V_{am}} \tag{15.212}$$

$$T_{in} = \frac{1}{2f_c} \tag{15.213}$$

where V_{dc} is the dc-link voltage input to the inverter, V_{cm} is the maximum control voltage, and f_c is the switching (carrier) frequency of the inverter.

The induced emf due to rotor flux linkages, e_a, is

$$e_a = \Psi_r \omega_r \tag{15.214}$$

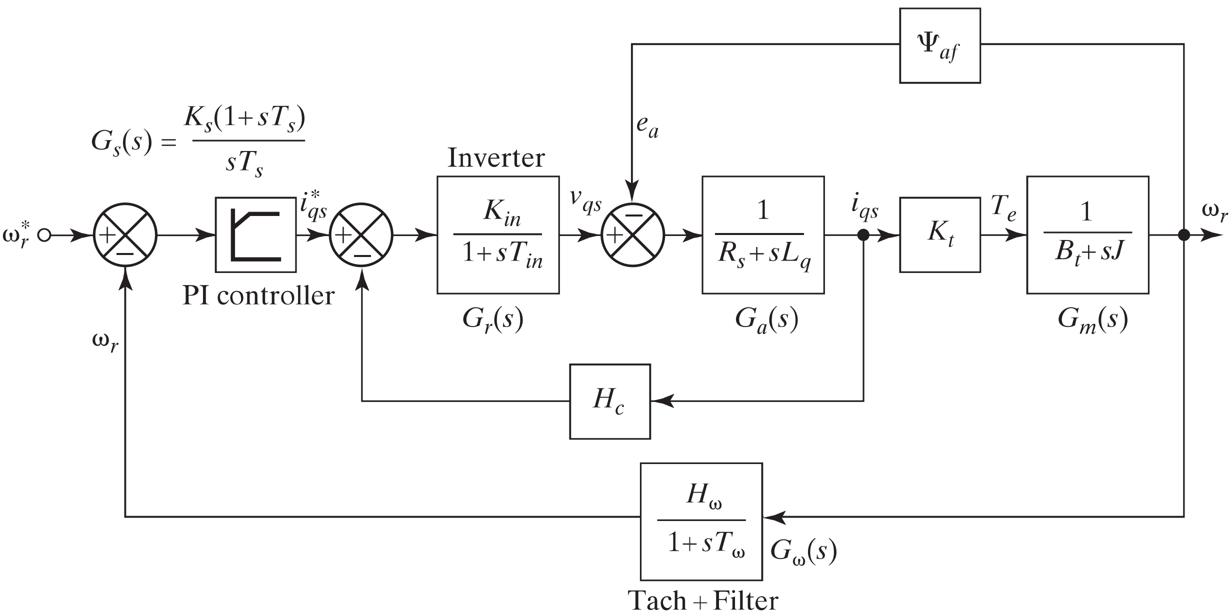


FIGURE 15.44

Block diagram of the speed-controlled drive system [23].

15.7.2 Current Loop

The induced-emf loop for Eq. (15.214), which crosses the q-axis current loop, could be simplified by moving the pick-off point for the induced-emf loop from speed to current output point. This is shown in Figure 15.45 and the simplified current-loop transfer function is given by

$$\frac{i_{qs}(s)}{i_{qs}^{*}(s)} = \frac{K_{in}K_a(1 + sT_m)}{H_cK_aK_{in}(1 + sT_m) + (1 + sT_{in})[K_aK_b + (1 + sT_a)(1 + sT_m)]} \tag{15.215}$$

where the constants are given by

$$K_a = \frac{1}{R_a}; \quad T_a = \frac{L_q}{R_s}; \quad K_m = \frac{1}{B_T}; \quad T_m = \frac{J}{B_T}; \quad K_b = K_T K_m \Psi_r \tag{15.216}$$

The following approximations near the vicinity of crossover frequency would simplify the design of the current and speed controllers.

$$1 + sT_{in} \cong 1$$

$$1 + sT_m \cong sT_m$$

$$(1 + sT_a)(1 + sT_m) \cong 1 + s(T_a + T_{in}) \cong 1 + sT_{ar}$$

where

$$T_{ar} = T_a + T_{in}$$

With these assumptions, the current-loop transfer function in Eq. (15.215) can be approximated as

$$\frac{i_{qs}(s)}{i_{qs}^{*}(s)} \cong \frac{(K_uK_{in}T_m)s}{K_aK_b + (T_m + K_aK_{in}T_mH_c)s + (T_mT_{ar})s^2} \cong \left(\frac{T_mK_{in}}{K_b}\right)\frac{s}{(1 + sT_1)(1 + sT_2)} \tag{15.217}$$

It is found in practical system that $T_1 < T_2 < T_{im}$, and thus $(1 + sT_2) \approx sT_2$. As a result, Eq. (15.217) can be further simplified to

$$\frac{i_{qs}(s)}{i_{qs}^{*}(s)} \cong \frac{K_i}{(1 + sT_i)} \tag{15.218}$$

where

$$K_i = \frac{T_mK_{in}}{T_2K_b} \tag{15.219}$$

$$T_i = T_1 \tag{15.220}$$

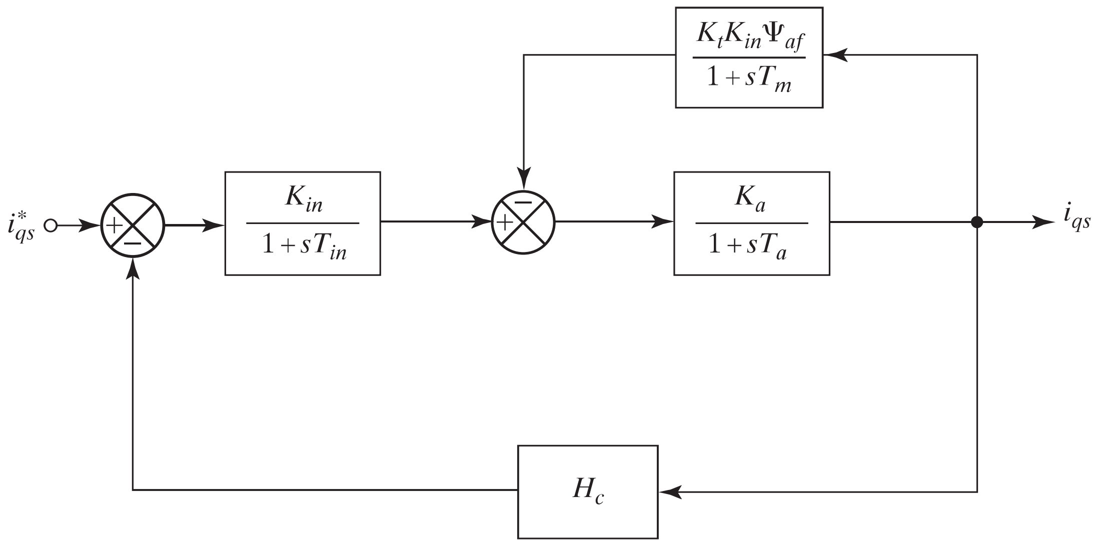


FIGURE 15.45

Block diagram of the current controller.

15.7.3 Speed Controller

The block diagram of the simplified current-loop transfer function with the speed loop is shown in Figure 15.46.

The following approximations at the vicinity of the crossover frequency can simplify the design of the speed controller.

$$1 + sT_m \cong sT_m$$
$$(1 + sT_i)(1 + sT_\omega) \cong 1 + sT_{\omega i}$$
$$1 + sT_\omega \cong 1$$

where

$$T_{\omega i} = T_\omega + T_i \tag{15.221}$$

With these approximations, the speed-loop transfer function is given by

$$GH(s) \cong \left(\frac{K_i K_m K_T H_\omega}{T_m}\right) \times \frac{K_s}{T_s} \times \frac{(1 + sT_s)}{(1 + sT_{\omega i})} \tag{15.222}$$

which can be used to obtain the closed-loop speed transfer function as given by

$$\frac{\omega_r(s)}{\omega_r^*(s)} = \frac{1}{H_\omega}\left[\frac{K_g \frac{K_s}{T_s}(1 + sT_s)}{s^3 T_{\omega i} + s^2 + K_g \frac{K_s}{T_s}(1 + sT_s)}\right] \tag{15.223}$$

where

$$K_g = \frac{K_i K_m K_T H_\omega}{T_m} \tag{15.224}$$

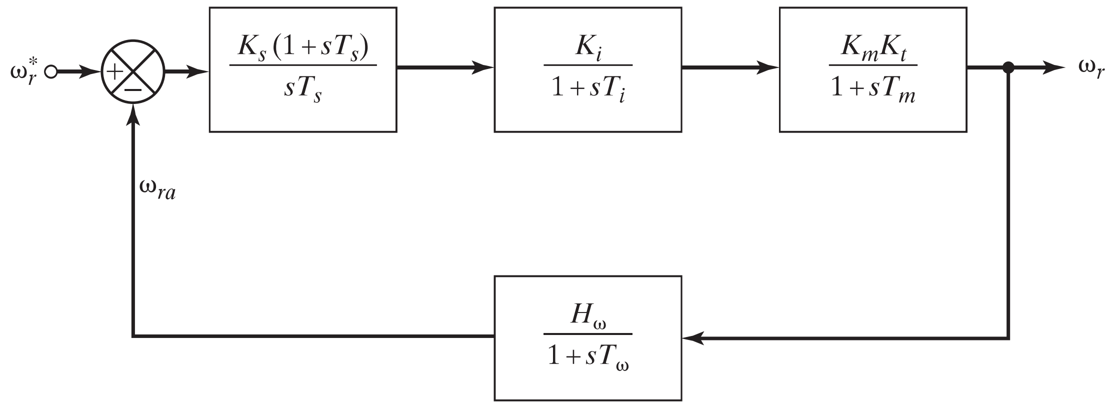


FIGURE 15.46

Simplified current-loop transfer function with the speed loop.

Equating this transfer function to a symmetric-optimum function with a damping ratio of 0.707 gives the closed-loop speed transfer function as

$$\frac{\omega_r(s)}{\omega_r^*(s)} \cong \frac{1}{H_\omega}\left[\frac{(1 + sT_s)}{\left(\frac{T_s^3}{16}\right)s^3 + \left(\frac{3T_s^2}{8}\right)s^2 + (T_s)s + 1}\right] \tag{15.225}$$

Equating the coefficients of Eqs. (15.223) and (15.225) and solving for the constants yields the time and gain constants of the speed controller as given by [23]

$$T_s = 6T_{\omega i} \tag{15.226}$$

$$K_s = \frac{4}{9K_gT_{\omega i}} \tag{15.227}$$

The proportional gain, K_{ps}, and integral gain, K_{is}, of the speed controller are derived as [23]

$$K_{ps} = K_s = \frac{4}{9K_gT_{\omega i}} \tag{15.228}$$

$$K_{is} = \frac{K_s}{T_s} = \frac{1}{27K_gT_{\omega i}^2} \tag{15.229}$$

Example 15.13 Finding the Speed Control Parameters of a PMSM Drive System

The parameters of a PMSM drive system are 220 V, Y-connected, 60 Hz, six-poles, $R_s = 1.2\,\Omega$, $L_d = 5\,\text{mH}$, $L_q = 8.4\,\text{mH}$, $\Psi_r = 0.14$ Wb-turn, $B_T = 0.01\,\text{N.mn/rad/s}$, $J = 0.006\,\text{kg-m}^2$, $f_c = 2.5\,\text{kHz}$, $V_{cm} = 10\,\text{V}$, $H_\omega = 0.05\,\text{V/V}$, $H_c = 0.8\,\text{V/A}$, $V_{dc} = 200\,\text{V}$.

Design an optimum-based speed controller for a damping ratio of 0.707.

Solution

$V_L = 220\,\text{V}$, $V_{ph} = V_L\sqrt{3} = 127\,\text{V}$, $f = 60\,\text{Hz}$, $p = 6$, $R_s = 1.2\Omega$, $L_d = 5\,\text{mH}$, $L_q = 8.4\,\text{mH}$, $\Psi_r = 0.14$ Wb-turn, $B_T = 0.01\,\text{N.mn/rad/s}$, $J = 0.006\,\text{kg-m}^2$, $f_c = 2.5\,\text{kHz}$, $V_{cm} = 10\,\text{V}$, $H_\omega = 0.05\,\text{V/V}$, $H_c = 0.8\,\text{V/A}$, $V_{dc} = 200\,\text{V}$.

Inverter gain from Eq. (15.212) $K_{in} = 0.65V_{dc}/V_{cm} = (0.65 \times 200)/10 = 13\,\text{V/V}$
Time constant from Eq. (15.213) $T_{in} = 1/(2f_c) = 1/(2 \times 2.5 \times 10^{-3}) = 0.2\,\text{ms}$
Therefore, the inverter transfer function is

$$G_r(s) = \frac{K_{in}}{1 + sT_{in}} = \frac{13}{1 + 0.0002s}$$

Motor electrical gain from Eq. (15.216) $K_a = 1/R_s = 1/1.2 = 0.8333\,\text{s}$
Motor time constant from Eq. (15.216) $T_a = L_q/R_s = 8.4 \times 10^{-3}/1.2 = 0.007\,\text{s}$
Therefore, the motor transfer function is

$$G_a(s) = \frac{K_a}{1 + sT_a} = \frac{0.8333}{1 + 0.007s}$$

Torque constant of the induced-emf loop from Eq. (15.210) is

$$K_T = \frac{3}{2} \times \left(\frac{p}{2}\right)^2 \Psi_r = \frac{3}{2} \times \left(\frac{6}{2}\right)^2 \times 0.14 = 1.89\ \text{N. m/A}$$

Mechanical gain from Eq. (15.216) $K_m = 1/B_T = 1/0.01 = 100\,\text{rad/s/N.m}$
Mechanical time constant from Eq. (15.216) $T_m = J/B_T = 0.006/0.01 = 0.6\,\text{s}$
Emf feedback constant from Eq. (15.216) $K_b = K_T K_m \Psi_r = 1.89 \times 100 \times 0.14 = 26.46$
Therefore, the emf feedback transfer function is

$$G_b(s) = \frac{K_b}{1 + sT_m} = \frac{26.46}{1 + 0.6s}$$

The motor mechanical transfer function is

$$G_m(s) = \frac{K_T K_m}{1 + sT_m} = \frac{1.89 \times 100}{1 + 0.6s} = \frac{189}{1 + 0.6s}$$

Electrical time constants of the motor can be solved from the roots of the following equation

$$as^2 + bs + c = 0$$

where $a = T_m(T_a + T_{in}) = 0.6 \times (0.007 + 0.2) = 0.004$

$b = T_m + K_a K_{in} T_m H_c = 0.6 + 0.8333 \times 13 \times 0.6 \times 0.8 = 5.8$

$c = K_a K_b = 0.8333 \times 26.46 = 22.05$

The inverse of the roots gives the time constants T_1 and T_2 as

$$\frac{1}{T_1} = -\frac{-b - \sqrt{b^2 - 4ac}}{2a} = -\frac{-5.8 - \sqrt{5.8^2 - 4 \times 0.004 \times 22.05}}{2 \times 22.05}; \quad T_1 = 0.7469\,\text{ms}$$

$$\frac{1}{T_2} = -\frac{-b + \sqrt{b^2 - 4ac}}{2a} = -\frac{-5.8 + \sqrt{5.8^2 - 4 \times 0.004 \times 22.05}}{2 \times 22.05}; \quad T_2 = 262.2916\,\text{ms}$$

Current-loop time constant from Eq. (15.220) $T_i = T_1 = 0.7469\,\text{ms}$

Current-loop gain from Eq. (15.219)

$$K_i = T_m K_{in}/(T_2 K_b) = 0.6 \times 13/(262.2916 \times 10^{-3} \times 26.46) = 1.12388$$

The simplified current-loop transfer functions from Eq. (15.218) is

$$G_{is}(s) = \frac{K_i}{(1 + sT_i)} = \frac{1.12388}{1 + 0.7469 \times 10^{-3}s}$$

The speed controller constant from Eq. (15.224) is

$$K_g = \frac{K_i K_m K_T H_\omega}{T_m} = \frac{1.12388 \times 100 \times 1.89 \times 0.05}{0.6} = 17.70113$$

The time constant from Eq. (15.221) $T_{\omega i} = T_\omega + T_i = 2\,\text{ms} + 0.7469\,\text{ms} = 2.7469\,\text{ms}$

The time constant from Eq. (15.226) $T_s = 6T_{\omega i} = 6 \times 2.7469\,\text{ms} = 16.48\,\text{ms}$

The gain constant from Eq. (15.227) $K_s = 4/(9K_g T_{\omega i}) = 4/(9 \times 17.70113 \times 2.7469\,\text{ms}) = 9.14042$.

The overall speed-loop transfer function is

$$G_{sp}(s) = \frac{G_m(s)G_i(s)G_s(s)}{1 + G_\omega(s)G_m(s)G_i(s)G_s(s)}$$

where

$$G_s(s) = \frac{K_s}{T_s}\frac{(1 + sT_s)}{s} = \frac{9.14042}{0.01648} \times \frac{(1 + 0.01648T_s)}{s} = 554.58 \times \frac{(1 + 0.01648T_s)}{s}$$

$$G_\omega(s) = \frac{H_\omega}{1 + sT_\omega} = \frac{0.05}{1 + 0.002s}$$

15.8 STEPPER MOTOR CONTROL

Stepper motors are electromechanical motion devices, which are used primarily to convert information in digital form to mechanical motion [19, 20]. These motors rotate at a predetermined angular displacement in response to a logic input. Whenever stepping from one position to another is required, the stepper motors are generally used. They are found as the drivers for the paper in line printers and in other computer peripheral equipment such as in positioning of the magnetic-disk head.

Stepper motors fall into two types: (1) the variable-reluctance stepper motor and (2) the permanent-magnet stepper motor. The operating principle of the variable-reluctance stepper motor is much the same as that of the synchronous reluctance machine, and the permanent-magnet stepper motor is similar in principle to the permanent-magnet synchronous machine.

15.8.1 Variable-Reluctance Stepper Motors

These motors can be used as a single unit or as a multistack. In the multistack operation, three or more single-phase reluctance motors are mounted on a common shaft with their stator magnetic axes displaced from each other. The rotor of a three-stack is shown in Figure 15.47. It has three cascaded two-pole rotors with a minimum-reluctance path

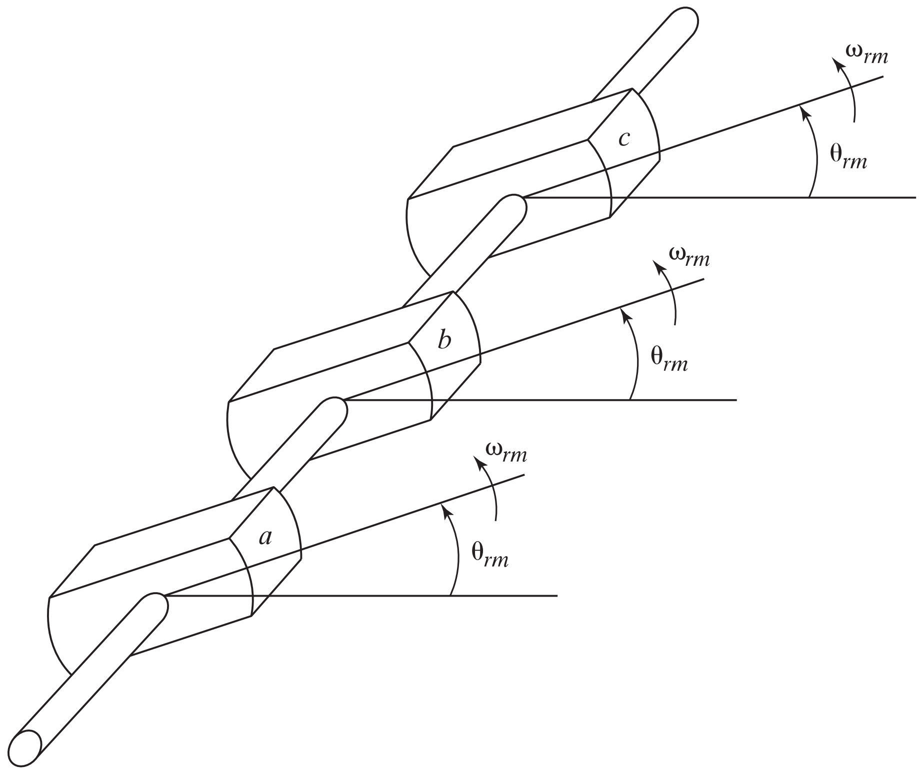


FIGURE 15.47

Rotor of two-pole, three-stack variable-reluctance stepper motors.

of each aligned at the angular displacement θ_{rm}. Each of these rotors has a separate, single-phase stator with the magnetic axes of the stators displaced from each other. The corresponding stators are shown in Figure 15.48.

Each stator has two poles with stator-winding wound around both poles. Positive current flows into as_1 and out as_1', which is connected to as_2 so that positive current flows into as_2 and out as_2'. Each winding can have several turns; and the number of turns from as_1 to as_1' is $NN_s/2$, which is the same as that from as_2 to as_2'. θ_{rm} is referenced from the minimum-reluctance path from the *as*-axis.

If the windings *bs* and *cs* are open circuited and the winding *as* is excited with a dc voltage, a constant current i_{as} can be immediately established. The rotor *a* can be aligned to the *as*-axis and $\theta_{rm} = 0$ or 180°. If the *as* winding is instantaneously de-energized and the *bs* winding is energized with a direct current, rotor *b* aligns itself with the minimum-reluctance path along the *bs*-axis. Thus, rotor *b* would rotate clockwise from $\theta_{rm} = 0$ to $\theta_{rm} = -60°$. However, instead of energizing the *bs* winding, if we energize the *cs* winding with a direct current, the rotor *c* aligns itself with the minimum-reluctance path along the *cs*-axis. Thus, the rotor *c* would rotate anticlockwise from $\theta_{rm} = 0$ to $\theta_{rm} = 60°$. Thus, applying a dc voltage separately in the sequences *as*, *bs*, *cs*, *as*, . . . produces 60° steps in the clockwise direction, whereas the sequence *as*, *cs*, *bs*, *as*, . . . produces 60° steps in the counterclockwise direction. We need at least three stacks to achieve rotation (stepping) in both directions.

If both the *as* and *bs* windings are energized at the same time, initially the *as* winding is energized with $\theta_{rm} = 0$ and the *bs* winding is energized without de-energizing the *as* winding. The rotor rotates clockwise from $\theta_{rm} = 0$ to $\theta_{rm} = -30°$. The step length is reduced by one-half. This is referred to as *half-step* operation. A stepper motor is a discrete device, operated by switching a dc voltage from one stator winding to the other.

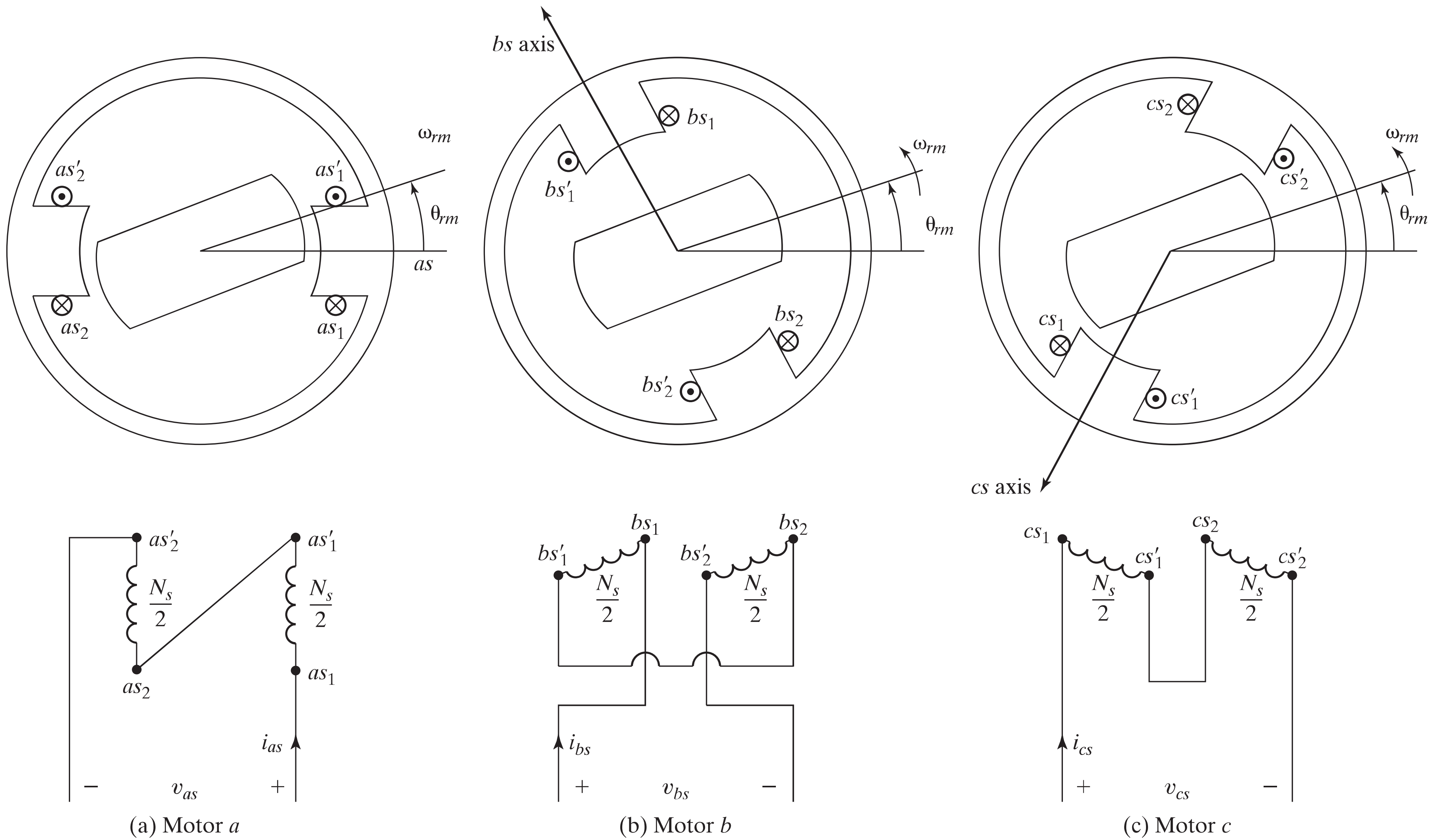


FIGURE 15.48

Stator configurations of two-pole, three-stack, variable-reluctance stepper motors.

Each stack is often called a *phase*. In other words, a three-stack machine is a three-phase machine. Although as many as seven stacks (phases) may be used, three-stack stepper motors are quite common.

The tooth pitch (T_p), which is the angular displacement between root teeth, is related to the rotor teeth per stack R_T by

$$T_p = \frac{2\pi}{R_T} \tag{15.230}$$

If we energize each stack separately, then going from *as* to *bs* to *cs* back to *as* causes the rotor to rotate one tooth pitch. If N is the number of stacks (phases), the step length S_L is related to T_p by

$$S_L = \frac{T_P}{N} = \frac{2\pi}{NR_T} \tag{15.231}$$

For $N = 3, R_T = 4, T_P = 2\pi/4 = 90°, S_L = 90°/3 = 30°$; and an *as*, *bs*, *cs*, *as*, ... sequence produces 30° steps in the clockwise direction. For $N = 3, R_T = 8, T_P = 2\pi/8 = 45°, S_L = 45°/3 = 15°$; and an *as*, *cs*, *bs*, *as*, ... sequence produces 15° steps in the counterclockwise direction. Therefore, by increasing the number of rotor teeth reduces the step length. The step lengths of multistack stepping motors typically range from 2° to 15°.

The torque developed by multistack stepper motors is given by

$$T_d = -\frac{R_T}{2} L_B[i_{as}^2 \sin(R_T\theta_{rm}) + i_{bs}^2 \sin(R_T(\theta_{rm} \pm S_L)) + i_{cs}^2 \sin(R_T(\theta_{rm} \pm S_L))] \tag{15.232}$$

The stator self-inductance varies with the rotor position, and L_B is the peak value at $\cos(p\theta_{rm}) = \pm 1$. Equation (15.232) can be expressed in terms of T_p as

$$\begin{aligned} T_d = -\frac{R_T}{2} L_B\Bigg[& i_{as}^2 \sin\left(\frac{2\pi}{T_p}\theta_{rm}\right) + i_{bs}^2 \sin\left(\frac{2\pi}{T_p}\left(\theta_{rm} \pm \frac{T_p}{3}\right)\right) \\ & + i_{cs}^2 \sin\left(\frac{2\pi}{T_p}\left(\theta_{rm} \pm \frac{T_p}{3}\right)\right)\Bigg] \end{aligned} \tag{15.233}$$

which indicates that the magnitude of the torque is proportional to the number of rotor teeth per stack R_T. The steady-state torque components in Eq. (15.232) against θ_{rm} are shown in Figure 15.49.

15.8.2 Permanent-Magnet Stepper Motors

The permanent-magnet stepper motor is also quite common. It is a permanent-magnet synchronous machine and it may be operated either as a stepping motor or as a continuous-speed device. However, we concern ourselves only with its application as a stepping motor.

The cross section of a two-pole, two-phase permanent-magnet stepper motor is shown in Figure 15.50. For explaining the stepping action, let us assume that the *bs*

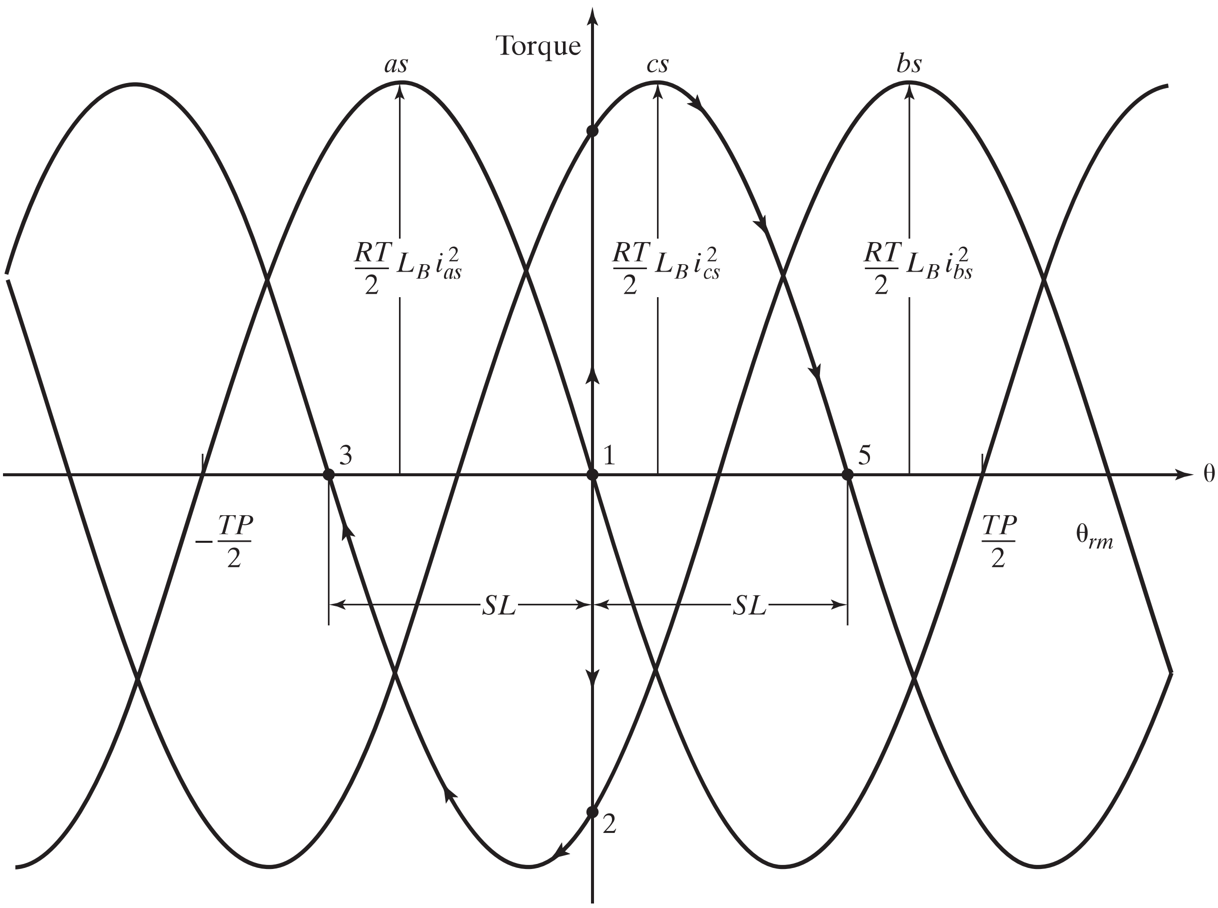
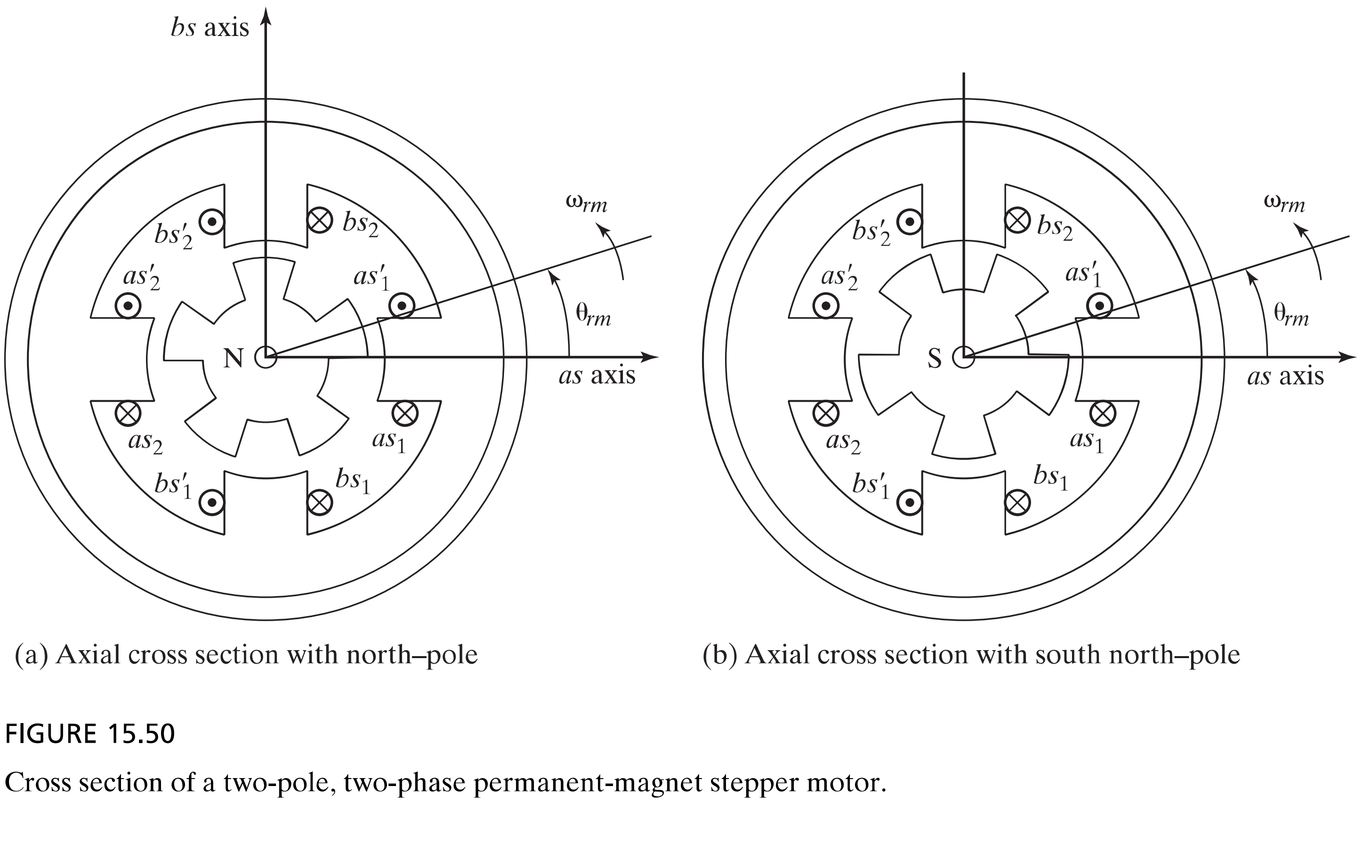


FIGURE 15.49

Steady-state torque components against the rotor angle θ_{rm} for a three-stack motor.



(a) Axial cross section with north–pole

(b) Axial cross section with south north–pole

FIGURE 15.50

Cross section of a two-pole, two-phase permanent-magnet stepper motor.

winding is open-circuited and apply a constant positive current through the *as* winding. As a result, this current establishes a stator south pole at the stator tooth on which the as_1 winding is wound, and stator north pole is established at the stator tooth on which the as_2 winding is wound. The rotor would be positioned at $\theta_{rm} = 0$. Now let us simultaneously de-energize the *as* winding while energizing the *bs* winding with a positive current. The rotor moves one step length in the counterclockwise direction. To continue stepping in the counterclockwise direction, the *bs* winding is de-energized and the *as* winding is energized with a negative current. That is, counterclockwise stepping occurs with a current sequence of $i_{as}, i_{bs} - i_{as}, -i_{bs}\, i_{as}, i_{bs} \ldots$. Clockwise rotation is achieved with a current sequence of $i_{as}, -i_{bs} - i_{as}, i_{bs}\, i_{as}, -i_{bs} \ldots$.

A counterclockwise rotation is achieved by a sequence of $i_{as}, i_{bs} - i_{as}, -i_{bs}\, i_{as}, i_{bs} \ldots$. Thus, it takes four switching (steps) to advance the rotor one tooth pitch. If N is the number of phases, the step length S_L is related to T_p by

$$S_L = \frac{T_P}{2N} = \frac{\pi}{N\,R_T} \tag{15.234}$$

For $N = 2$, $R_T = 5$, $T_P = 2\pi/5 = 72°$, $S_L = 72°/(2 \times 2) = 18°$. Therefore, increasing the number of phases and rotor teeth reduces the step length. The step lengths typically range from 2° to 15°. Most permanent-magnet stepper motors have more than two poles and more than five rotor teeth; some may have as many as eight poles and as many as 50 rotor teeth.

The torque developed by a permanent-magnet stepper motor is given by

$$T_d = -R_T \lambda'_m [i_{as} \sin(R_T \theta_{rm}) - i_{bs} \sin(R_T \theta_{rm})] \tag{15.235}$$

where λ'_m is the amplitude of the flux linkages established by the permanent magnet as viewed from the stator phase windings. It is the constant inductance times a constant current. In other words, the magnitude of λ'_m is proportional to the magnitude of the open-circuit sinusoidal voltage induced in each stator phase winding.

The plots of the torque components in Eq. (15.235) are shown in Figure 15.51. The term $\pm T_{d(am)}$ is the torque due to the interaction of the permanent magnet and $\pm i_{as}$, and the term $\pm T_{d(bm)}$ is the torque due to the interaction of the permanent magnet and $\pm i_{bs}$. The reluctance of the permanent magnet is large, approaching that of air. Because the flux established by the phase currents flows through the magnet, the reluctance of the flux path is relatively large. Hence, the variation in the reluctance due to rotation of the rotor is small and, consequently, the amplitudes of the reluctance torques are small relative to the torque developed by the interaction between the magnet and the phase currents. For these reasons, the reluctance torques are generally neglected.

Notes:

1. For a permanent-magnet stepper motor, it is necessary for the phase currents to flow in both directions to achieve rotation. For a variable-reluctance stepper motor, it is not necessary to reverse the direction of the current in the stator

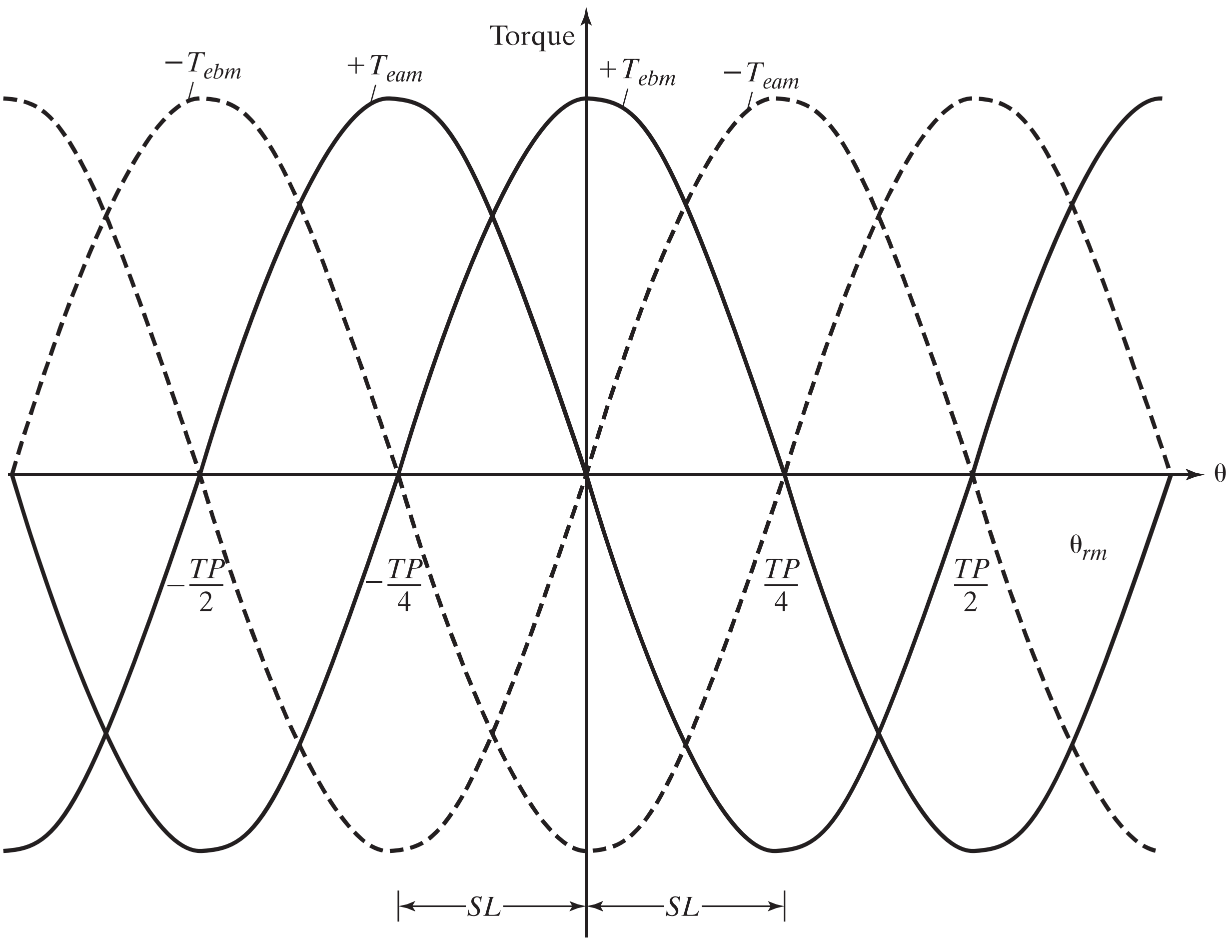


FIGURE 15.51

Steady-state torque components against the rotor angle θ_{rm} for a permanent-magnet stepper motor with constant phase currents.

windings to achieve rotation and, therefore, the stator voltage source need only be unidirectional.

2. Generally, stepper motors are supplied from a dc voltage source; hence, the power converter between the phase windings and the dc source must be bidirectional; that is, it must have the capability of applying a positive and negative voltage to each phase winding.
3. Permanent-magnet stepper motors are often equipped with what is referred to as *bifilar* windings. Instead of only one winding on each stator tooth, there are two identical windings with one wound opposite to the other, each having separate independent external terminals. With this type of winding configuration the direction of the magnetic field established by the stator windings is reversed, not by changing the direction of the current but by reversing the sense of the winding through which current is flowing. This will, however, increase the size and weight of the stepper motor.
4. Hybrid stepper motors [21] whose construction is a hybrid between permanent magnet and reluctance motor topologies broaden the range of applications and offer enhanced performance with simpler and lower cost of power converters.

Key Points of Section 15.8

- Stepper motors are electromechanical motion devices, which are used primarily to convert information in digital form to mechanical motion. Stepper motors are synchronous machines that are operated as stepping motors.
- Stepper motors fall into two types: the variable-reluctance stepper motor and the permanent-magnet stepper motor. A variable-reluctance stepper motor requires only unidirectional current flow whereas a permanent-magnet stepper motor requires bidirectional flow unless the motor has *bifilar* stator windings.

15.9 LINEAR INDUCTION MOTORS

The linear induction motors find industrial applications in high-speed ground transportation, sliding door systems, curtain pullers, and conveyors [24]. An induction motor has a circular motion whereas a linear motor has a linear motion. If an induction motor is cut and laid flat, it would be like a linear motor. The stator and rotor of the rotating motor correspond to the primary and secondary sides, respectively, of the linear induction motor. The primary side consists of a magnetic core with a three-phase winding. The secondary side may be either metal sheet or a three-phase winding wound around a magnetic core. A linear induction motor has an open-ended air-gap and magnetic structure owing to the finite lengths of the primary and secondary sides.

A linear induction motor may be single sided or double sided, as shown in Figure 15.52. In order to reduce the total reluctance of the magnetic path in a single-sided linear induction motor with a metal sheet as the secondary winding, as shown in Figure 15.52a, the metal sheet is backed by a ferromagnetic material such as iron. When a supply voltage is applied to the primary winding of a three-phase linear induction motor, the magnetic field produced in the air-gap region travels at the synchronous speed. The interaction of the magnetic field with the induced currents in the secondary exerts a thrust on the secondary to move in the same direction if

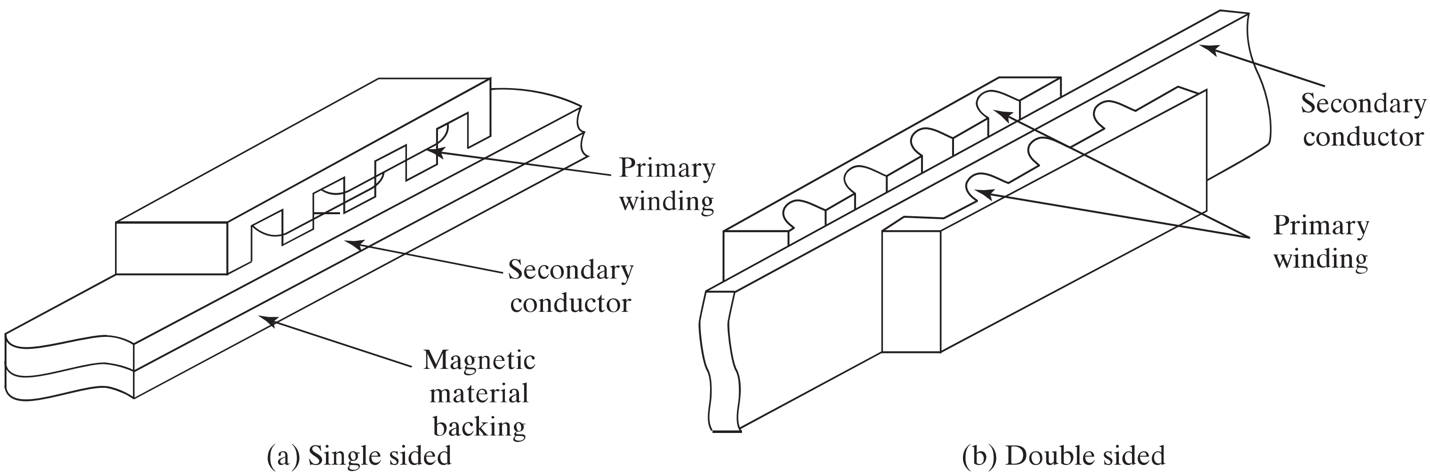


FIGURE 15.52

Cross sections of linear induction motors.

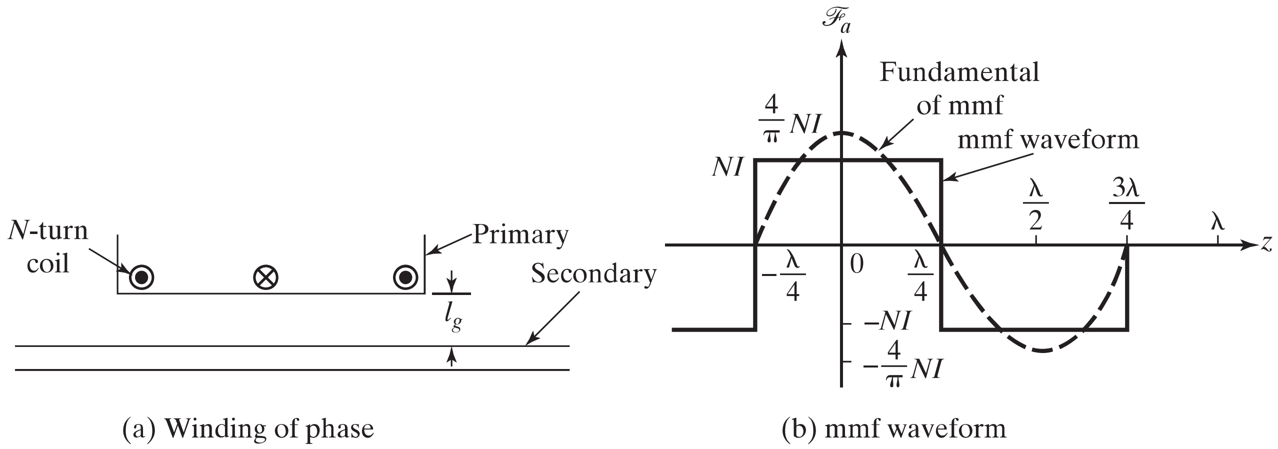


FIGURE 15.53

Schematic of a phase winding and its mmf waveform.

the primary is held stationary. On the other hand, if the secondary side is stationary and the primary is free to move, the primary will move in the direction opposite to that of the magnetic field. In order to maintain a constant thrust (force) over a considerable distance, one side is kept shorter than the other. For example, in high-speed ground transportation, a short primary and a long secondary are being used. In such a system, the primary is an integral part of the vehicle, whereas the track acts as the secondary.

Let us consider only one phase winding, say phase A, of the three-phase primary winding, as shown in Figure 15.53a. The N-turn phase winding experiences an mmf of NI, as shown in Figure 15.53b, and the fundamental of the mmf waveform is given by

$$\Im_a = k_\omega \frac{2}{n\pi} N i_a \cos\left(\frac{2\pi}{\lambda} z\right) \tag{15.236}$$

where

k_ω = the winding factor
i_a = the instantaneous value of the fundamental current in phase a
λ = the wavelength of the field which equals to the winding pitch
n = the number of periods over the length of the motor
z = an arbitrary location in the linear motor

Each phase winding is displaced from the others by a distance of $\pi/3$ and excited by a balanced three-phase supply of angular frequency ω. Thus the net mmf in the motor consists of only a forward-traveling wave component as given by

$$\Im(z, t) = \frac{3}{2} F_m \cos\left(\omega t - \frac{2\pi}{\lambda} z\right) \tag{15.237}$$

where

$$F_m = \frac{2}{n\pi} k_\omega N i_a \tag{15.238}$$

The synchronous velocity of the traveling mmf can be determined by setting the argument of the cosine term of Eq. (15.237) to a constant value C as given by

$$\omega t - \frac{2\pi}{\lambda} z = C \tag{15.239}$$

This can be differentiated to give the linear velocity as

$$V_s = \frac{dz}{dt} = \frac{\omega \lambda}{2\pi} = \lambda f \tag{15.240}$$

where f is the operating frequency of the supply. Equation (15.240) can also be expressed in terms of the pole pitch τ as

$$V_s = 2\tau f \tag{15.241}$$

Thus, the synchronous velocity v_s is independent of the number of poles in the primary winding, and the number of poles need not be an even number. The slip of a linear induction motor is defined as

$$s = \frac{v_s - v_m}{v_s} \tag{15.242}$$

where v_m is the linear velocity of the motor. The power and thrust in a linear induction motor can be calculated by using the equivalent circuit of an induction motor. Thus, using Eq. (15.9), we get the air-gap power, P_g, as

$$P_g = 3I_2^2 \frac{r_2}{s} \tag{15.243}$$

and the developed power, P_d, is

$$P_d = (1 - s)P_g \tag{15.244}$$

and the developed thrust, F_d, is

$$F_d = \frac{P_d}{v_m} = \frac{P_g}{v_s} = 3I_2^2 \frac{r_2}{sv_s} \tag{15.245}$$

The velocity–thrust characteristic of a linear induction motor is similar to the speed–torque characteristic of a conventional induction motor. The velocity in linear induction motor decreases rapidly with the increasing thrust, as shown in Figure 15.54. For this reason these motors often operate with low slip, leading to a relatively low efficiency.

A linear induction motor displays a phenomenon known as end effects because of its open-ended construction. There are two end effects: static and dynamic. The static end effect occurs solely because of the asymmetric geometry of the primary. This results in asymmetric flux distribution in the air-gap region and gives rise to unequal induced voltages in the phase windings. The dynamic end effect occurs as a result of the relative motion of the primary side with respect to the secondary. The conductor coming under

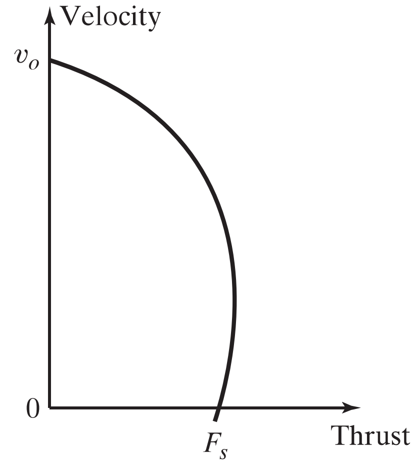


FIGURE 15.54
Typical velocity versus thrust characteristic.

the leading edge opposes the magnetic flux in the air gap, while the conductor leaving the trailing edge tries to sustain the flux. Therefore, the flux distribution is distorted and the increased losses in the secondary side reduce the efficiency of the motor.

Example 15.14 Finding the Developed Power by the Linear Inductor Motor

The parameters of a linear induction motors are pole pitch $= 0.5\,\text{m}$, supply frequency $f = 60\,\text{Hz}$. The speed of the primary side is 210 km/h, and the developed thrust is 120 kN. Calculate (a) the motor speed v_{m} (b) the developed power P_d, (c) the synchronous speed v_{s}, (d) the slip, and (e) the copper loss in the secondary P_{cu}.

Solution

$\lambda = 0.5\,\text{m}$, $f = 60\,\text{Hz}$, $v = 210\,\text{km/h}$, $F_a = 120 \times 10^3\,\text{N}$

a. Motor speed $v_m = v/3600 = 210 \times 10^3/3600 = 58.333\,\text{m/s}$
b. Developed power $P_d = F_a v_m = 120 \times 10^3 \times 58.333 = 7\,\text{MW}$
c. Synchronous speed $v_s = 2\lambda f = 2 \times 0.5 \times 60 = 60\,\text{m/s}$
d. Slip $s = (v_s - v_m)/v_m = (60 - 58.333)/60 = 0.028$
e. Copper loss $P_{cu} = F_a s v_s = 120 \times 10^3 \times 0.028 \times 60 = 200\,\text{kW}$

Key Points of Section 15.9

- A linear motor has a linear motion whereas an induction motor has a circular motion.
- The stator and rotor of the rotating motor correspond to the primary and secondary sides, respectively, of the linear induction motor.

15.10 HIGH-VOLTAGE IC FOR MOTOR DRIVES

Power electronics plays a key role in modern motor drives, requiring high-performance advanced control techniques along with other start-up and protection functions. The features include gate driving with protection, soft-start charging of the dc bus and linear current sensing of the motor phase current, and control algorithms from volt or hertz to sensorless vector or servo control. The block diagram of a typical drive

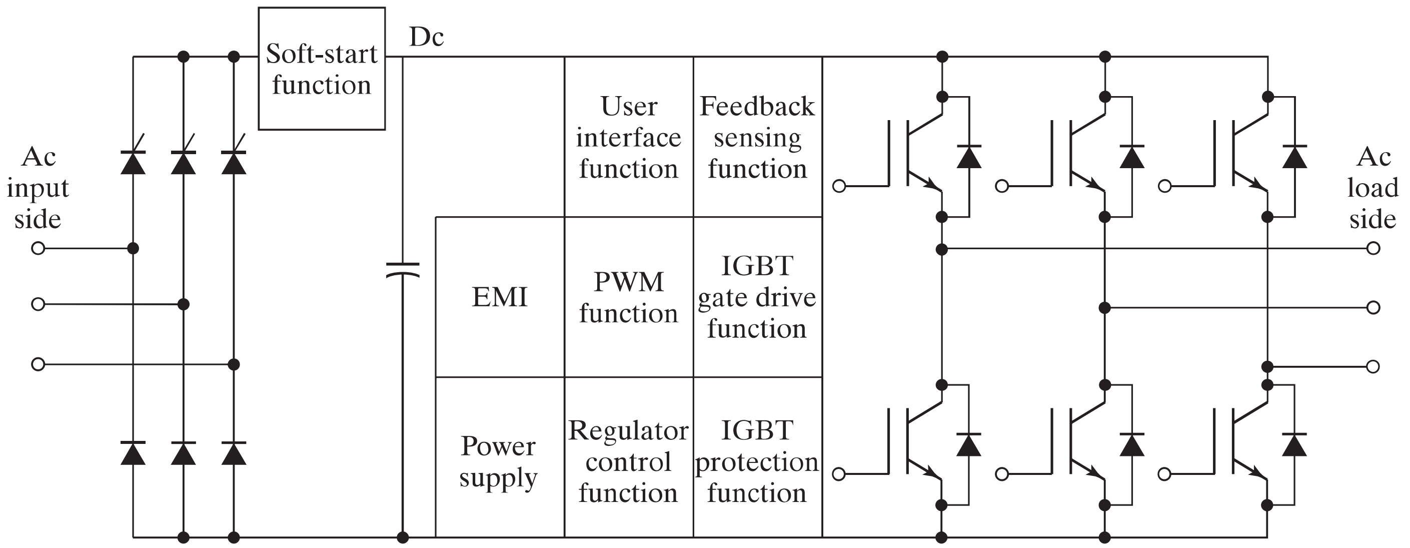


FIGURE 15.55

Functional block diagram of an inverter-fed drive, Ref. 25. (Courtesy of International Rectifier, Inc.)

and its associated functions are shown in Figure 15.55 [25]. Each function serves its unique requirements but also has to couple with each other for the complete system to work as a whole. For example, the IGBT gate drive and protection functions have to be synchronized, and the feedback sensing and the regulator control and PWM have to be matched.

The motor drives require functions such as protection and soft shutdown for the inverter stage, current sensing, analog-to-digital conversion for use in the algorithm for closed-loop current control, soft charge of the dc bus capacitor, and an almost bulletproof input converter stage. The simplicity and cost are important factors for applications such as in refrigerator compressors, air-conditioner compressors, and direct drive washing machines.

The market demands for industrial motor drives, home alliance, and light industrial drives have led to the development of high-voltage ICs for motor drives known as *power conversion processors* (PCPs) by the power device manufacturer [25]. The motor drive IC family, which is the monolithic integration of high-voltage circuits with the gate drive, enables power conversion with advanced control features to meet the needs of high-performance drives with ruggedness, compact size, and lower electromagnetic interference (EMI). The architecture of the IC family can be classified into three types: (1) two-level power conversion processing, (2) single-level power conversion processing, and (3) mixed-mode power conversion processing.

Two-level power conversion processing. The signal-processing functions are implemented within an isolated low-voltage supply level that is remote from the power level. All power devices are contained within the high-voltage supply level directly connected to the ac main. Different types of technologies are then used to connect the two levels. Gate drives are supplied through optocouplers, feedback functions

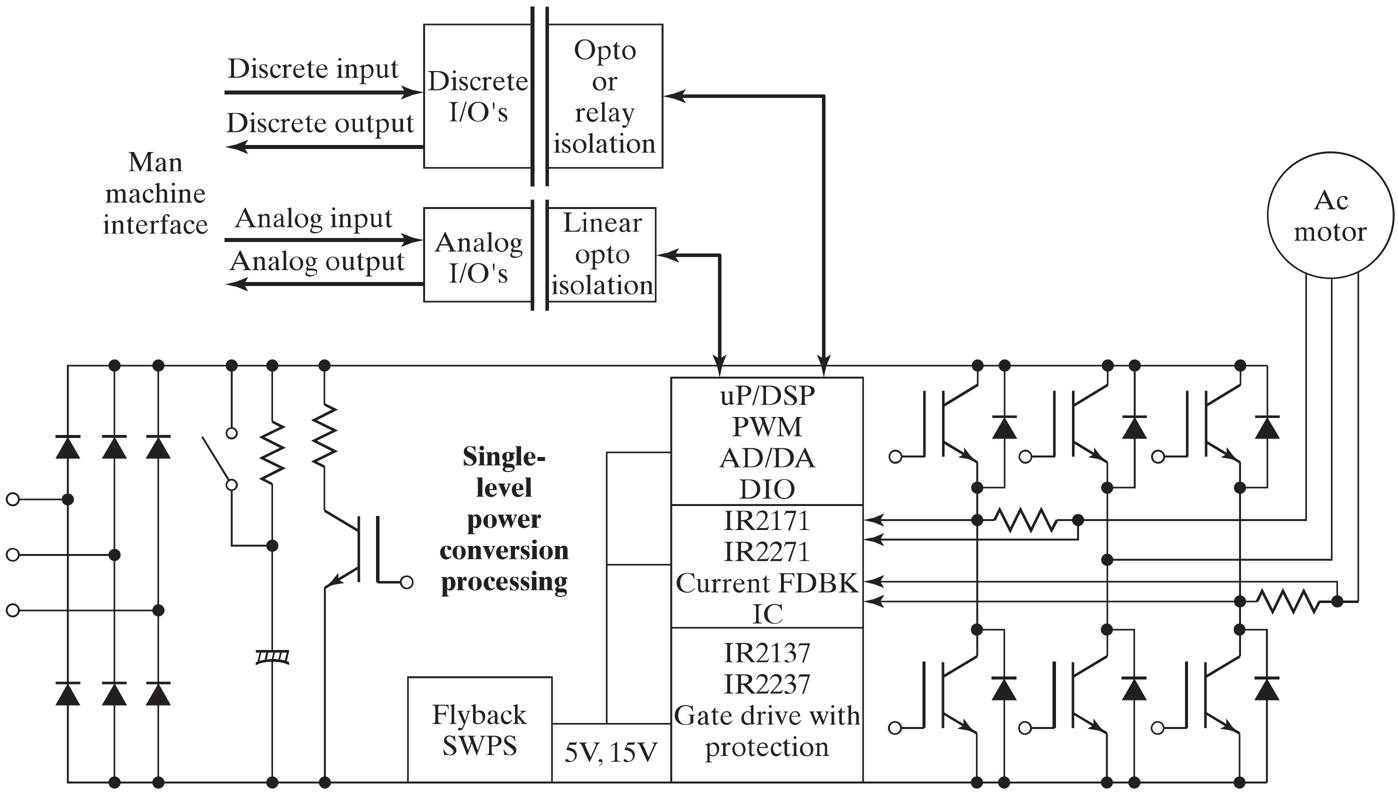


FIGURE 15.56

Two-level power conversion-processing architecture, Ref. 25. (Courtesy of International Rectifier, Inc.)

are implemented through combination of linear optocouplers and hall-effect sensors, and soft-start function is implemented through relay. A bulky multiple-winding transformer is also needed to supply the various isolated supplies for the different functions. This type of architecture, as shown in Figure 15.56, is being phased out.

Single-level power conversion processing. All gate drive, protection, feedback sensing, and control functions are implemented within the same level of the high-voltage supply rail, and all the functions are coupled with each other in the same electrically connected level. Protection is localized and more effective. The board layout is more compact, contributing to lower EMI and lower cost of the total system. This type of architecture (as shown in Figure 15.57) is compact and most effective for special-purpose drives such as for home appliances and small industrial drives less than 3.75 kW; these are called *microinverters* (or *microdrives*).

Mixed-mode power conversion processing. The power conversion processing is done primarily at the high-voltage supply level. A second level of signal processing is used for motion profiling and communication. This second level helps facilitate network and option card connections for general-purpose drives. Also, it simplifies the encoder connection for position sensing in servo drives. The two levels of processing are connected through an isolated serial bus. This type of architecture is shown in Figure 15.58. A comparison between the different power conversion architectures is listed in Table 15.1.

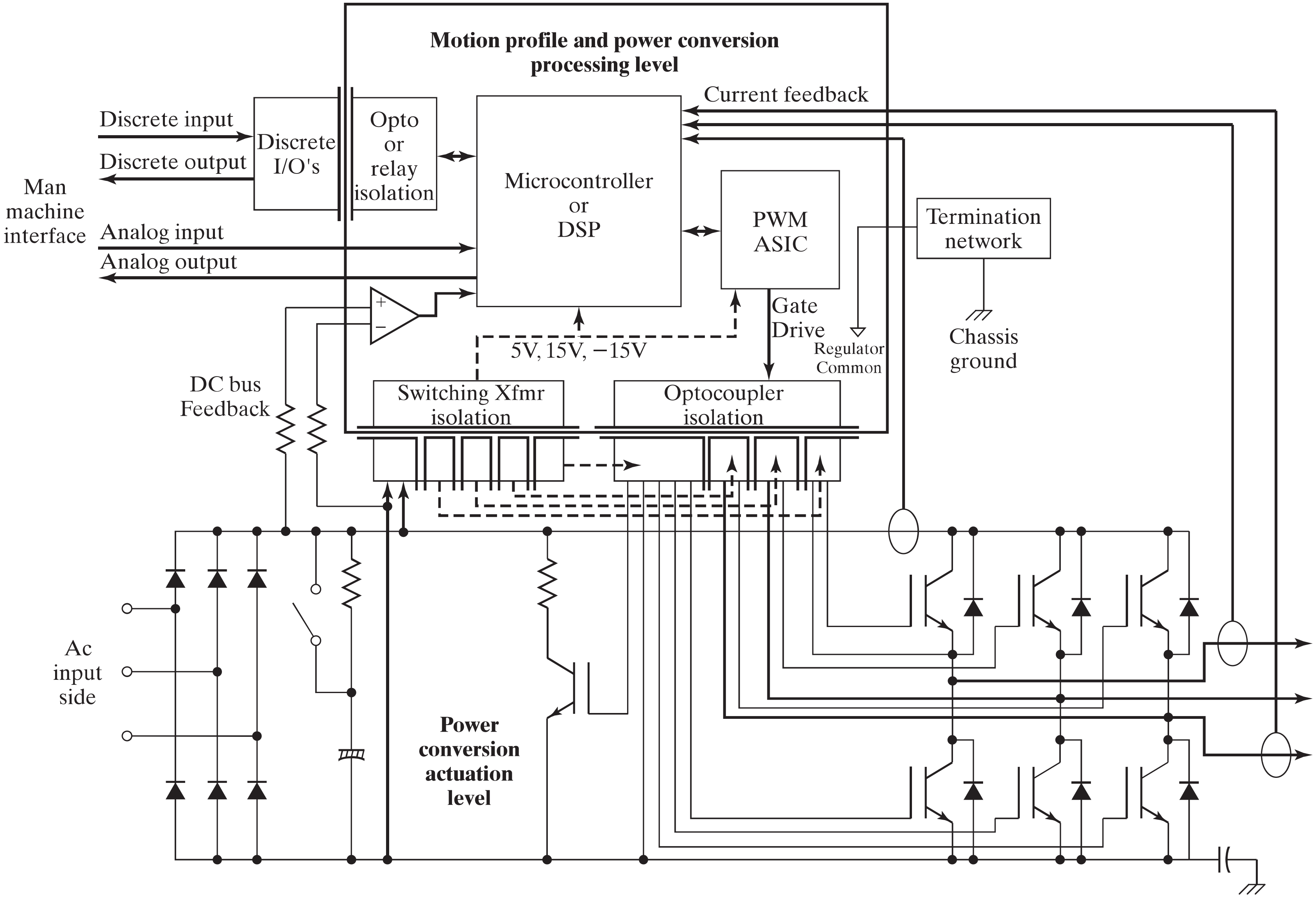


FIGURE 15.57

Single-level power conversion-processing architecture with an input-side diode rectifier, Ref. 25. (Courtesy of International Rectifier, Inc.)

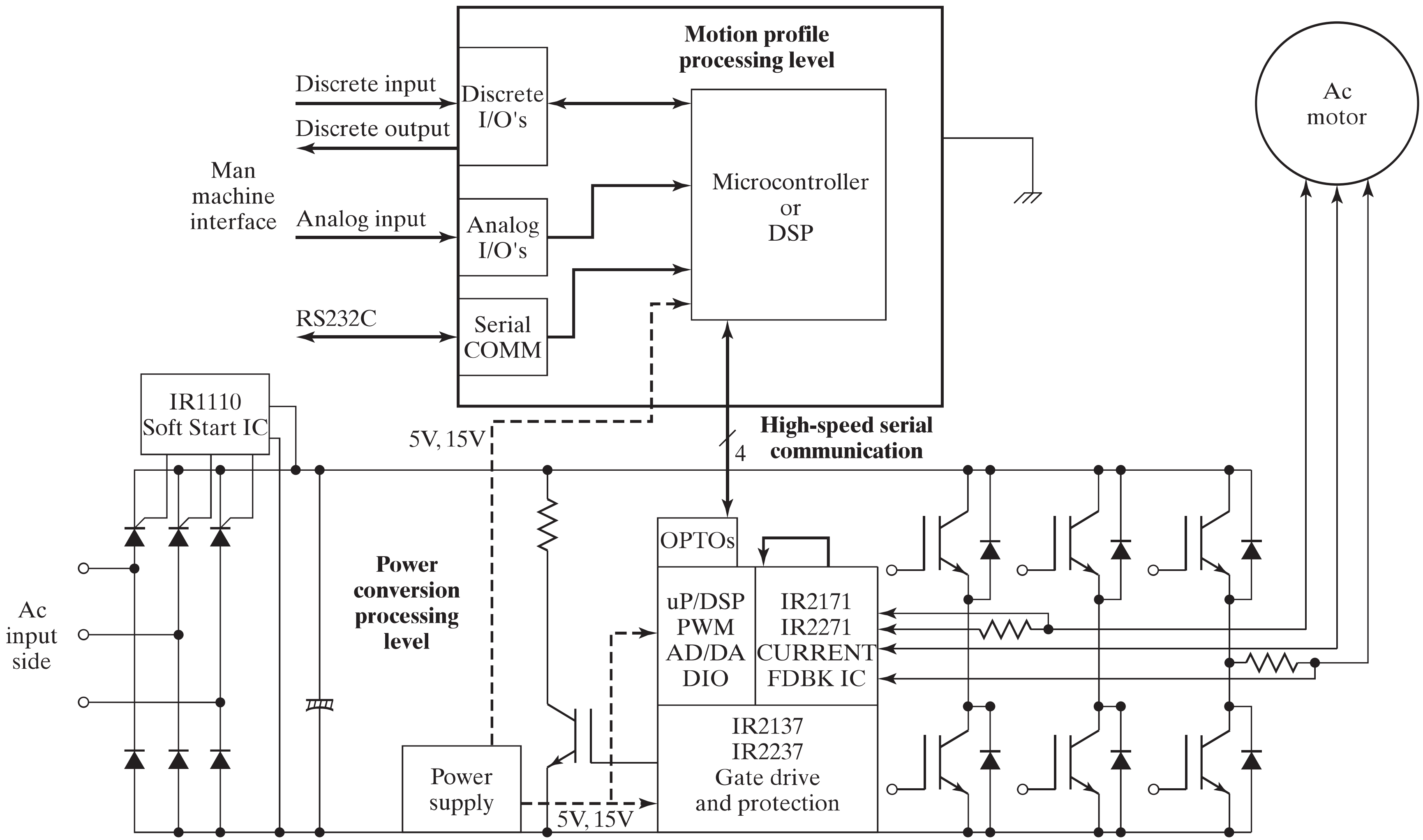


FIGURE 15.58

Single-level power conversion-processing architecture with an input-side controlled rectifier, Ref. 25. (Courtesy of International Rectifier, Inc.)

TABLE 15.1 Comparison of Two-Level versus Single-Level Power Conversion Architecture

Two-Level Architecture	Single-Level Architecture
Motion and power conversion processed together	Motion and power conversion processed separately
Isolation by optodrivers (sensitive high-speed signals)	Isolation by digital interface (high noise-margin signals)
Large dead time	Small dead time
Complex switching supply	Simple flyback supply
Large hall current sensors	Small HVIC current sensors
Protection at signal level	Protection at power level
Larger size and more EMI	Smaller size and less EMI

One of the key features of the single-level and mixed-mode power conversion-processing architecture is the integration of the gate drive, protection, and sensing functions. The integration is implemented in a high-voltage integrated circuit (HVIC) technology. Multifunction sensing chips that integrate current and voltage feedback with both amplitude and phase information can simplify the designs of ac or brushless dc (BLDC) motor drives. Monolithic integration of the gate drive, protection, linear current sensing, and more functions in a single piece of silicon using HVIC technology is the ultimate goal. Thus, all power conversion functions for robust, efficient, cost-effective, compact motor drives should ideally be integrated in modular fashion with appropriately defined serial communication protocol for local or remote control.

Key Points of Section 15.10

- An IC gate drive integrates most of the control functions including some protection functions to operate under overload and fault conditions. There are numerous IC gate drives that are commercially available for gating power converters.
- The special-purpose ICs for motor drives include many features such as gate driving with protection, soft-start charging of dc bus, linear current sensing of motor phase current, and control algorithms from volts-hertz to sensor-less vector or servo control.

SUMMARY

Although ac drives require advanced control techniques for control of voltage, frequency, and current, they have advantages over dc drives. The voltage and frequency can be controlled by voltage-source inverters. The current and frequency can be controlled by current-source inverters. The slip power recovery schemes use controlled rectifiers to recover the slip power of induction motors. The most common method of closed-loop control of induction motors is volts/hertz, flux, or slip control. Both squirrel-cage and wound-rotor motors are used in variable-speed drives. A voltage-source inverter can supply a number of motors connected in parallel, whereas a current-source inverter can supply only one motor.

Synchronous motors are constant-speed machines and their speeds can be controlled by voltage, frequency, or current. Synchronous motors are of six types: cylindrical rotors, salient poles, reluctance, permanent magnets, switched reluctance motors, and brushless dc and ac motors. Whenever stepping from one position to another is required, the stepper motors are generally used. Synchronous motors can be operated as stepper motors. Stepper motors fall into two types: the variable-reluctance stepper motor and the permanent-magnet stepper motor. Due to the pulsating nature of the converter voltages and currents, it requires special specifications and design of motors for variable-speed applications [22]. There is abundant literature on ac drives, so only the fundamentals are covered in this chapter.

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REVIEW QUESTIONS

15.1 What are the types of induction motors?
15.2 What is a synchronous speed?
15.3 What is a slip of induction motors?
15.4 What is a slip frequency of induction motors?
15.5 What is the slip at starting of induction motors?
15.6 What are the torque–speed characteristics of induction motors?
15.7 What are various means for speed control of induction motors?
15.8 What are the advantages of volts/hertz control?
15.9 What is a base frequency of induction motors?
15.10 What are the advantages of current control?
15.11 What is a scalar control?
15.12 What is a vector control?
15.13 What is an adaptive control?
15.14 What is a static Kramer drive?
15.15 What is a static Scherbius drive?
15.16 What is a field-weakening mode of induction motor?
15.17 What are the effects of frequency control of induction motors?
15.18 What are the advantages of flux control?
15.19 How can the control characteristic of an induction motor be made to behave like a dc motor?
15.20 What are the various types of synchronous motors?
15.21 What is the torque angle of synchronous motors?
15.22 What are the differences between salient-pole motors and reluctance motors?
15.23 What are the differences between salient-pole motors and permanent-magnet motors?
15.24 What is a pull-out torque of synchronous motors?
15.25 What is the starting torque of synchronous motors?
15.26 What are the torque–speed characteristics of synchronous motors?
15.27 What are the V-curves of synchronous motors?
15.28 What are the advantages of voltage-source inverter-fed drives?
15.29 What are the advantages and disadvantages of reluctance motor drives?
15.30 What are the advantages and disadvantages of permanent-magnet motors?
15.31 What is a switched reluctance motor?
15.32 What is a self-control mode of synchronous motors?
15.33 What is a brushless dc motor?
15.34 What is a brushless ac motor?
15.35 What is a stepper motor?

15.36 What are the types of stepper motors?

15.37 What are the differences between variable reluctance and permanent-magnet stepper motors?

15.38 How is the step of a variable reluctance stepper motor controlled?

15.39 How is the step of a permanent-magnet stepper motor controlled?

15.40 Explain the different speeds and their relationships to each other—the supply speed ω, the rotor speed ω_r, the mechanical speed ω_m, and the synchronous speed ω_s.

15.41 What is the difference between an induction motor and a linear induction motor?

15.42 What are the end effects of linear induction motors?

15.43 What is the purpose of dimensioning the control variables?

15.44 What is the purpose of making the damping factor close to 0.707 while designing a controller for a motor drive?

PROBLEMS

15.1 A three-phase, 460-V, 60-Hz, eight-pole, Y-connected induction motor has $R_s = 0.08\ \Omega$, $R_r' = 0.1\ \Omega$, $X_s = 0.62\ \Omega$, $X_r' = 0.92\ \Omega$, and $X_m = 6.7\ \Omega$. The no-load loss, $P_{\text{no load}} =$ 300 W. At a motor speed of 750 rpm, use the approximate equivalent circuit in Figure 15.2 to determine **(a)** the synchronous speed ω_s; **(b)** the slip s; **(c)** the input current I_i; **(d)** the input power P_i; **(e)** the input power factor of the supply, PF_s; **(f)** the gap power P_g; **(g)** the rotor copper loss P_{ru}; **(h)** the stator copper loss P_{su}; **(i)** the developed torque T_d; **(j)** the efficiency; **(k)** the starting rotor current I_{rs} and the starting torque T_s; **(l)** the slip for maximum torque s_m; **(m)** the maximum motoring developed torque T_{mm}; and **(n)** the maximum regenerative developed torque T_{mr}.

15.2 Repeat Problem 15.1 if R_s is negligible.

15.3 Repeat Problem 15.1 if the motor has two poles and the parameters are $R_s = 1.02\ \Omega$, $R_r' = 0.35\ \Omega$, $X_s = 0.72\ \Omega$, $X_r' = 1.08\ \Omega$, and $X_m = 60\ \Omega$. The no-load loss is $P_{\text{no load}} =$ 70 W and the rotor speed is 3250 rpm.

15.4 The parameters of an induction motor are 2000 hp, 2300 V, three phase, star-connected, four poles, 60 Hz, full load slip = 0.03746, $R_s = 0.02\,\Omega$, $R_r' = 0.12\,\Omega$, $R_m = 45\,\Omega$, $X_m = 50\,\Omega$, $X_s = X_r' = 0.32\,\Omega$. Find **(a)** the efficiency of an induction motor operating at full load and **(b)** the per-phase capacitance required to obtain a line power factor of unity by installing capacitors at the input terminals of the induction motor.

15.5 The parameters of an induction motor are 20 hp, 230 V, three phase, star-connected, four poles, 50 Hz, full load slip = 0.03746, $R_s = 0.02\,\Omega$, $R_r' = 0.12\,\Omega$, $R_m = 45\,\Omega$, $X_m = 50\,\Omega$, $X_s = X_r' = 0.32\,\Omega$. Find **(a)** the efficiency of an induction motor operating at full load and **(b)** the per-phase capacitance required to obtain a line power factor of unity by installing capacitors at the input terminals of the induction motor.

15.6 A three-phase, 460-V, 60-Hz, six-pole Y-connected induction motor has $R_s = 0.32\ \Omega$, $R_r' = 0.18\ \Omega$, $X_s = 1.04\ \Omega$, $X_r' = 1.6\ \Omega$, and $X_m = 18.8\ \Omega$. The no-load loss, $P_{\text{no load}}$, is negligible. The load torque, which is proportional to speed squared, is 180 N·m at 1180 rpm. If the motor speed is 850 rpm, determine **(a)** the load torque demand T_L; **(b)** the rotor current I_r'; **(c)** the stator supply voltage V_a; **(d)** the motor input current I_i; **(e)** the motor input power P_i; **(f)** the slip for maximum current s_a; **(g)** the maximum rotor current $I_{r(\max)}$; **(h)** the speed at maximum rotor current ω_a; and **(i)** the torque at the maximum current T_a.

15.7 Repeat Problem 15.6 if R_s is negligible.

15.8 Repeat Problem 15.6 if the motor has four poles and the parameters are $R_s = 0.25\ \Omega$, $R_r' = 0.14\ \Omega$, $X_s = 0.7\ \Omega$, $X_r' = 1.05\ \Omega$, and $X_m = 20.6\ \Omega$. The load torque is 121 N·m at 1765 rpm. The motor speed is 1425 rpm.

15.9 A three-phase, 460-V, 60-Hz, six-pole, Y-connected wound-rotor induction motor whose speed is controlled by slip power, as shown in Figure 15.7b, has the following parameters: $R_s = 0.11\ \Omega$, $R'_r = 0.09\ \Omega$, $X_s = 0.4\ \Omega$, $X'_r = 0.6\ \Omega$, and $X_m = 11.6\ \Omega$. The turns ratio of the rotor to stator windings is $n_m = N_r/N_s = 0.9$. The inductance L_d is very large and its current, I_d, has negligible ripple. The values of R_s, R'_r, X_s, and X'_r for the equivalent circuit in Figure 15.2 can be considered negligible compared with the effective impedance of L_d. The no-load loss is 275 W. The load torque, which is proportional to speed squared, is 455 N · m at 1175 rpm. **(a)** If the motor has to operate with a minimum speed of 850 rpm, determine the resistance R. With this value of R, if the desired speed is 950 rpm, calculate **(b)** the inductor current I_d, **(c)** the duty-cycle k of the dc converter, **(d)** the dc voltage V_d, **(e)** the efficiency, and **(f)** the input PF_s of the drive.

15.10 Repeat Problem 15.9 if the minimum speed is 650 rpm.

15.11 Repeat Problem 15.9 if the motor has eight poles and the motor parameters are $R_s = 0.08\ \Omega$, $R'_r = 0.1\ \Omega$, $X_s = 0.62\ \Omega$, $X'_r = 0.92\ \Omega$, and $X_m = 6.7\ \Omega$. The no-load loss is $P_{\text{no load}} = 300$ W. The load torque, which is proportional to speed, is 604 N · m at 785 rpm. The motor has to operate with a minimum speed of 650 rpm, and the desired speed is 750 rpm.

15.12 A three-phase, 460-V, 60-Hz, six-pole, Y-connected wound-rotor induction motor whose speed is controlled by a static Kramer drive, as shown in Figure 15.7b, has the following parameters: $R_s = 0.11\ \Omega$, $R'_r = 0.09\ \Omega$, $X_s = 0.4\ \Omega$, $X'_r = 0.6\ \Omega$, and $X_m = 11.6\ \Omega$. The turns ratio of the rotor to stator windings is $n_m = N_r/N_s = 0.9$. The inductance L_d is very large and its current, I_d, has negligible ripple. The values of R_s, R'_r, X_s, and X'_r for the equivalent circuit in Figure 15.2 can be considered negligible compared with that of the effective impedance of L_d. The no-load loss is 275 W. The turns ratio of the converter ac voltage to supply voltage is $n_c = N_a/N_b = 0.5$. If the motor is required to operate at a speed of 950 rpm, calculate **(a)** the inductor current I_d, **(b)** the dc voltage V_d, **(c)** the delay angle α of the converter, **(d)** the efficiency, and **(e)** the input PF_s of the drive. The load torque, which is proportional to speed squared, is 455 N · m at 1175 rpm.

15.13 Repeat Problem 15.12 for $n_c = 0.9$.

15.14 For Problem 15.12 plot the power factor against the turns ratio n_c.

15.15 A three-phase, 56-kW, 3560-rpm, 460-V, 60-Hz, two-pole, Y-connected induction motor has the following parameters: $R_s = 0$, $R_r = 0.18\ \Omega$, $X_s = 0.13\ \Omega$, $X_r = 0.2\ \Omega$, and $X_m = 11.4\ \Omega$. The motor is controlled by varying the supply frequency. If the breakdown torque requirement is 170 N · m, calculate **(a)** the supply frequency, and **(b)** the speed ω_m at the maximum torque. Use the rated power and the speed to calculate T_{mb}.

15.16 If $R_s = 0.07\ \Omega$ and the frequency is changed from 60 to 40 Hz in Problem 15.15, determine the change in breakdown torque.

15.17 The motor in Problem 15.15 is controlled by a constant volts to hertz ratio corresponding to the rated voltage and rated frequency. Calculate the maximum torque T_m, and the corresponding speed ω_m for supply frequency of **(a)** 60 Hz, and **(b)** 30 Hz.

15.18 Repeat Problem 15.17 if R_s is 0.2 Ω.

15.19 A three-phase, 30-hp, 1780-rpm, 460 V, 60-Hz, four-pole, Y-connected induction motor has the following parameters: $R_s = 0.30$, $R'_r = 0.20\ \Omega$, $X_m = 30\ \Omega$, $X_s = 0.6\ \Omega$, $X_r' = 0.83\ \Omega$, The no-load loss is negligible. The motor is controlled by a current-source inverter and the input current is maintained constant at 50 A. If the frequency is 40 Hz and the developed torque is 220 N · m, determine **(a)** the slip for maximum torque s_m and maximum torque T_m, **(b)** the slip s, **(c)** the rotor speed ω_m, **(d)** the terminal voltage per phase V_a, and **(e)** the PF_m.

15.20 Repeat Problem 15.19 if frequency is 50 Hz.

15.21 The motor parameters of a volts/hertz inverter-fed induction motor drive are 8 hp, 240 V, 60 Hz, three phase, Y-connected, four poles, 0.88 PF and 90% efficiency, $R_s = 0.30$, $R_r' = 0.20\,\Omega$, $X_m = 30\,\Omega$, $X_s = 0.6\,\Omega$, and $X_r' = 0.83\,\Omega$. Find **(a)** the maximum slip speed **(b)** the rotor voltage drop V_o, **(c)** the volt-hertz constant K_{vf}, and **(d)** the dc-link voltage in terms of stator frequency f.

15.22 The motor parameters of a volts/hertz inverter-fed induction motor drive are 6 hp, 200 V, 60 Hz, three phase, Y-connected, four poles, 0.88 PF and 90% efficiency, $R_s = 0.30$, $R_r' = 0.20\,\Omega$, $X_m = 30\,\Omega$, $X_s = 0.6\,\Omega$, $X_r' = 0.83\,\Omega$. Find **(a)** the maximum slip speed **(b)** the rotor voltage drop V_o, **(c)** the volt-hertz constant K_{vf}, and **(d)** the dc-link voltage in terms of stator frequency f.

15.23 The motor parameters of a volts/hertz inverter-fed induction motor drive are 8 hp, 240 V, 60 Hz, three phase, Y-connected, four poles, 0.86 PF and 90% efficiency, $R_s = 0.30$, $R_r' = 0.20\,\Omega$, $X_m = 30\,\Omega$, $X_s = 0.6\,\Omega$, $X_r' = 0.83\,\Omega$. Find **(a)** the constants K^*, K_{tg}, K_f, and **(b)** express the rectifier output voltage v_r in terms of slip frequency ω_{sl} if the rated mechanical speed is $N = 1780$ rpm and $V_{cm} = 10\,\text{V}$.

15.24 The motor parameters of a volts/hertz inverter-fed induction motor drive are 6 hp, 200 V, 60 Hz, three phase, Y-connected, four poles, 0.86 PF and 84% efficiency, $R_s = 0.30$, $R_r' = 0.20\,\Omega$, $X_m = 30\,\Omega$, $X_s = 0.6\,\Omega$, $X_r' = 0.83\,\Omega$. Find **(a)** the constants K^*, K_{tg}, K_f, and **(b)** express the rectifier output voltage v_r in terms of slip frequency ω_{sl} if the rated mechanical speed is $N = 1780\,\text{rpm}$ and $V_{cm} = 10\,\text{V}$.

15.25 The parameters of an induction motor are 5 hp, 220 V, Y-connected, three-phase, 60 Hz, four poles, $R_s = 0.28\,\Omega, R_r = 0.18\,\Omega, L_m = 54\,\text{mH}, L_s = 5\,\text{mH}, L_r = 56\,\text{mH}$, and stator to rotor turns ratio, $a = 3$. The motor is supplied with its rated and balanced voltages. Find **(a)** the q- and d-axis steady-state voltages and currents, and **(b)** phase currents i_{qr}, i_{dr}, i_α, and i_β when the rotor is locked. Use the stator-reference-frames model of the induction machine.

15.26 The parameters of an induction motor with an indirect vector controller are 8 hp, Y-connected, three phase, 60 Hz, four poles, 240 V, $R_s = 0.28\,\Omega$, $R_r = 0.17\,\Omega$, $L_m = 61\,\text{mH}$, $L_r = 56\,\text{mH}$, $L_s = 53\,\text{mH}$, $J = 0.01667\,\text{kg-m}^2$, and rated speed $= 1800\,\text{rpm}$. Find **(a)** the rated rotor flux linkages and the corresponding stator currents i_{ds} and i_{qs}, **(b)** the total stator current I_s, **(c)** the torque angle θ_T, and **(d)** the slip gain K_{sl}.

15.27 The parameters of an induction motor with an indirect vector controller are 4 hp, Y-connected, three phase, 60 Hz, four poles, 240 V, $R_s = 0.28\,\Omega$, $R_r = 0.17\,\Omega$, $L_m = 61\,\text{mH}$, $L_r = 56\,\text{mH}$, $L_s = 53\,\text{mH}$, $J = 0.01667\,\text{kg-m}^2$, and rated speed $= 1800\,\text{rpm}$. Find **(a)** the rated rotor flux linkages and the corresponding stator currents i_{ds} and i_{qs}, **(b)** the total stator current I_s, **(c)** the torque angle θ_T, and **(d)** the slip gain K_{sl}.

15.28 A three-phase, 460-V, 60-Hz, 8-pole, Y-connected cylindrical rotor synchronous motor has a synchronous reactance of $X_s = 0.6\ \Omega$ per phase and the armature resistance is negligible. The load torque, which is proportional to the speed squared, is $T_L = 1200\ \text{N}\cdot\text{m}$ at 720 rpm. The power factor is maintained at 0.88 lagging by field control and the voltage- to-frequency ratio is kept constant at the rated value. If the inverter frequency is 40 Hz and the motor speed is 680 rpm, calculate **(a)** the input voltage V_a, **(b)** the armature current I_a, **(c)** the excitation voltage V_f, **(d)** the torque angle δ, and **(e)** the pull-out torque T_p.

15.29 A three-phase, 230-V, 60-Hz, 45-kW, eight-pole, Y-connected salient-pole synchronous motor has $X_d = 3.5\ \Omega$ and $X_q = 0.3\ \Omega$. The armature resistance is negligible. If the motor operates with an input power of 25 kW at a leading power factor of 0.88, determine **(a)** the torque angle δ, **(b)** the excitation voltage V_f, and **(c)** the torque T_d.

15.30 A three-phase, 230-V, 60-Hz, 8-pole, Y-connected reluctance motor has $X_d = 18\ \Omega$ and $X_q = 3.5\ \Omega$. The armature resistance is negligible. The load torque, which is proportional to speed, is $T_L = 15\ \text{N} \cdot \text{m}$. The voltage-to-frequency ratio is maintained constant at the rated value. If the supply frequency is 60 Hz, determine **(a)** the torque angle δ, **(b)** the line current I_a for a PF of 0.65 lagging, and **(c)** the pullout torque.

15.31 The parameters of a PMSM drive system are 240 V, Y-connected, 60 Hz, six poles, $R_s = 1.4\,\Omega$, $L_d = 6\,\text{mH}, L_q = 9\,\text{mH}$, $\Psi_r = 0.15\,\text{Wb-turn}$, $B_T = 0.01\,\text{N.mn/rad/s}$, $J = 0.006\,\text{kg-m}^2$, $J = 0.006\,\text{kg-m}^2$, $f_c = 2\,\text{kHz}, V_{cm} = 10\,\text{V}$, $H_\omega = 0.05\,\text{V/V}$, $H_c = 0.8\,\text{V/A}$, $T_\omega = 2$ ms, and $V_{\text{dc}} = 240\,\text{V}$. Design an optimum-based speed controller for a damping ratio of 0.707.

15.32 A three-phase, 230-V, 60-Hz, 8-pole, Y-connected reluctance motor has $X_d = 18\ \Omega$ and $X_q = 3.5\ \Omega$. The armature resistance is negligible. The load torque, which is proportional to speed, is $T_L = 15\ \text{N} \cdot \text{m}$. $T_L = 3.5\ \Omega$. The voltage-to-frequency ratio is maintained constant at the rated value. If the supply frequency is 60 Hz, determine **(a)** the torque angle δ,, **(b)** the line current I_a for a PF of 0.65 lagging, and **(c)** the pullout torque.

15.33 A variable reluctance stepper motor has six stacks and the step length $S_L = 12°$. The step sequence is *as, bs, cs, as, . . .* Find **(a)** the tooth pitch T_P, and **(b)** the number of rotor teeth per stack.

15.34 A variable reluctance stepper motor has three stacks and the step length $S_L = 12°$. The step sequence is *as, cs, bs, as, . . .* Find **(a)** the tooth pitch T_P, and **(b)** the number of rotor teeth per stack.

15.35 A four-pole permanent-magnet stepper motor, which rotates in the clockwise direction, has two stacks and the step length $S_L = 12°$. Find **(a)** the tooth pitch T_P, and **(b)** the number of rotor teeth per stack.

15.36 The parameters of a linear induction motor are synchronous speed = 72 m/s, supply frequency $f = 60\,\text{Hz}$. The speed of the primary side is 150 km/h, and the developed thrust is 100 kN. Calculate **(a)** the motor speed v_{m}, **(b)** the developed power P_d, **(c)** the pole pitch λ, **(d)** the slip, and **(e)** the copper loss in the secondary P_{cu}.

15.37 The parameters of a linear induction motor are synchronous speed = 96 m/s, supply frequency $f = 60\,\text{Hz}$. The speed of the primary side is 200 km/h, and the developed thrust is 250 kN. Calculate **(a)** the motor speed v_{m}, **(b)** the developed power P_d, **(c)** the pole pitch λ, **(d)** the slip, and **(e)** the copper loss in the secondary P_{cu}.